Ⓗ = Horner

# Fractions algébriques : Solutions

1. Simplifier les fractions suivantes après avoir préciser les conditions d'existence :

   (a) $\frac{x^2-4}{x+2}$ CE : $x \neq -2$

   $= \frac{(x-2)\cancel{(x+2)}}{\cancel{x+2}} = x-2$

   (b) $\frac{x^2-6x+9}{x-3}$ CE : $x \neq 3$

   $= \frac{(x-3)^{\cancel{2}}}{\cancel{(x-3)}} = x-3$

   (c) $\frac{x+3}{2x^2+x-15}$ CE Ⓗ : $x \neq -3$ , $x \neq \frac{5}{2}$

   $= \frac{\cancel{x+3}}{\cancel{(x+3)}(2x-5)} = \frac{1}{2x-5}$

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(d) $\dfrac{2x^2+3x-9}{x^2+x-6}$

(M)

$\underline{CE}$ : $x \neq 2$, $x \neq -3$

(M)

$= \dfrac{\cancel{(x+3)}(2x-3)}{(x-2)\cancel{(x+3)}} = \dfrac{2x-3}{x-2}$

(e) $\dfrac{x^2+5x+4}{3x^2+10x-8}$

(M)

$\underline{CE}$ : $x \neq -4$, $x \neq \frac{2}{3}$

(M)

$= \dfrac{(x+1)\cancel{(x+4)}}{\cancel{(x+4)}(3x-2)} = \dfrac{x+1}{3x-2}$

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(f) $\dfrac{x^2-4}{2x+x^2}$

CE: $x \neq 0$, $x \neq -2$

$$= \frac{(x-2)\cancel{(x+2)}}{x\cancel{(2+x)}} = \frac{x-2}{x}$$

(g) $\dfrac{x^2+3}{x^2}$

CE $x \neq 0$

$$= \frac{x^2+3}{x^2}$$

(h) $\dfrac{x^3+5x^2+6x}{x^3+2x^2-3x}$

(H) CE: $x \neq 0$, $x \neq 1$, $x \neq -3$

(H)
$$= \frac{\cancel{x}(x+2)\cancel{(x+3)}}{\cancel{x}(x-1)\cancel{(x+3)}} = \frac{x+2}{x-1}$$

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2. Simplifier les fractions suivantes après avoir préciser les conditions d'existence :

(a) $\frac{x}{x+1} + \frac{-3x}{x-2}$ CE : $x \neq -1$, $x \neq 2$

$$= \frac{x^2 - 2x - 3x^2 - 3x}{(x+1)(x-2)} = \frac{-x^2 - 5x}{(x+1)(x-2)}$$

(b) $\frac{x-1}{x+1} - \frac{x+1}{x-1}$ CE : $x \neq \pm 1$

$$= \frac{(x-1)^2 - (x+1)^2}{(x-1)(x+1)} = \frac{x^2 - 2x + 1 - x^2 - 2x - 1}{(x-1)(x+1)} = \frac{-4x}{(x-1)(x+1)}$$

(c) $\frac{3}{x-2} - \frac{2}{2x+5}$ CE : $x \neq 2$ ; $x \neq -\frac{5}{2}$

$$= \frac{3(2x+5) - 2(x-2)}{(x-2)(2x+5)} = \frac{6x + 15 - 2x + 4}{(x-2)(2x+5)} = \frac{4x + 19}{(x-2)(2x+5)}$$

(d) $\frac{5x+1}{x-1} + \frac{3}{2(x-1)}$ CE : $x \neq 1$

$$= \frac{2(5x+1) + 3}{2(x-1)} = \frac{10x + 2 + 3}{2(x-1)} = \frac{10x + 5}{2(x-1)}$$

(e) $\frac{3x}{x^2-1} - \frac{4}{x+1}$ CE : $x \neq \pm 1$

$$= \frac{3x - 4(x-1)}{x^2 - 1} = \frac{-x + 4}{x^2 - 1}$$

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(f) $\frac{5}{x^2-2x+1} - \frac{2}{x^2-4x+4} = \frac{5}{(x-1)^2} - \frac{2}{(x-2)^2}$

CE : $x \neq 1$
$x \neq 2$

$$= \frac{5(x^2-4x+4) - 2(x^2-2x+1)}{(x-1)^2(x-2)^2}$$

$$= \frac{3x^2-16x+18}{(x-1)^2(x-4)^2}$$

(g) $\frac{x}{x-1} - \frac{x}{x+1} + \frac{2x^2}{x^2-1}$

CE : $x \neq \pm 1$

$$= \frac{x(x+1) - x(x-1) + 2x^2}{(x-1)(x+1)}$$

$$= \frac{2x^2+2x}{(x-1)(x+1)} = \frac{2x\cancel{(x+1)}}{(x-1)\cancel{(x+1)}} = \frac{2x}{x-1}$$

(h) $\frac{3x-1}{x^2-2x-8} - \frac{2}{x+2} = \frac{3x-1}{(x-4)(x+2)} - \frac{2}{(x+2)}$

CE : $x \neq 4$
$x \neq -2$

$$= \frac{3x-1-2(x-4)}{(x-4)(x+2)} = \frac{x+7}{(x-4)(x+2)}$$

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3. Simplifier les fractions suivantes après avoir préciser les conditions d'existence :

(a) $\dfrac{x+3}{x^2+3x+2} \cdot \dfrac{x+2}{x^2+7x+12}$

CE (M) : $x \neq -2$, $x \neq -1$
$x \neq -3$, $x \neq -4$

(M)

$$= \frac{\cancel{x+3}}{(x+1)\cancel{(x+2)}} \cdot \frac{\cancel{x+2}}{\cancel{(x+3)}(x+4)}$$

$$= \frac{1}{(x+1)(x+4)}$$

(b) $\dfrac{x+3}{x-5} \cdot \dfrac{x^2-3x-10}{x^2+4x+3}$

(M)

CE : $x \neq 5$, $x \neq -3$, $x \neq -1$

(M)

$$= \frac{\cancel{x+3}}{\cancel{x-5}} \; \frac{\cancel{(x-5)}(x+2)}{\cancel{(x+3)}(x+1)} = \frac{x+2}{x+1}$$

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(c) $\frac{x^2-x}{x^2-1}.\frac{x^2+3x+2}{x^2-4}$

CE : $x \neq \pm 1$ , $x \neq \pm 2$

(H)

$$= \frac{x\cancel{(x-1)}}{\cancel{(x-1)}\cancel{(x+1)}} \cdot \frac{\cancel{(x+1)}\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}$$

$$= \frac{x}{x-2}$$

(d) $\frac{2x^2-7x-15}{3x^2-15x}.\frac{x^2}{2x^2+3x}$

(H)

$$= \frac{\cancel{(x-5)}\cancel{(2x+3)}}{3\cancel{x}\cancel{(x-5)}} \cdot \frac{\cancel{x^2}}{\cancel{x}\cancel{(2x+3)}}$$

CE : $x \neq 0$, $x \neq 5$

$x \neq -\frac{3}{2}$

$$= \frac{1}{3}$$

(e) $\frac{x+1}{x^2+3x} \div \frac{x^2+2x+1}{x^2+4x+3}$ $= \frac{x+1}{x^2+3x} \cdot \frac{x^2+4x+3}{x^2+2x+1}$

CE : $x \neq 0$ , $x \neq -3$, $x \neq -1$

(H)

$$= \frac{\cancel{x+1}}{x\cancel{(x+3)}} \cdot \frac{\cancel{(x+1)}\cancel{(x+3)}}{\cancel{(x+1)^2}} = \frac{1}{x}$$

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(f) $\dfrac{x+2}{2x-1} \div \dfrac{3x^2+4x-4}{2x^2+5x-3}$

M

$= \dfrac{\cancel{x+2}}{\cancel{2x-1}} \cdot \dfrac{(x+3)\cancel{(2x-1)}}{\cancel{(x+2)}(3x-2)}$

CE : $x \neq \frac{1}{2}$, $x \neq -2$, $x \neq \frac{2}{3}$

$= \dfrac{x+3}{3x-2}$

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4. Résoudre les équations fractionnaires suivantes après avoir préciser les conditions d'existence :

(a) $\frac{x-1}{x+5} - 4 = 0$

CE : $x \neq -5$

$\Leftrightarrow \frac{x-1-4(x+5)}{\cancel{x+5}} = 0$

$\Leftrightarrow -3x - 21 = 0$

$\Leftrightarrow x = -7$

$S : \{-7\}$

(b) $\frac{2x-8}{3x^2} = 0$

CE $x \neq 0$

$\Leftrightarrow 2x - 8 = 0$

$\Leftrightarrow x = 4$

$S : \{4\}$

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(c) $\frac{3(x-1)}{2x-3} = 1$ CE $x \neq \frac{3}{2}$

$$\Leftrightarrow \frac{3(x-1)}{2x-3} - 1 = 0$$

$$\Leftrightarrow \frac{3x - 3 - 2x + 3}{\cancel{2x-3}} = 0 \Leftrightarrow x = 0 \qquad S: \{0\}$$

(d) $\frac{1}{x} + \frac{1}{2} = 2$ CE : $x \neq 0$

$$\Leftrightarrow \frac{2 + x}{\cancel{2x}} = \frac{4x}{\cancel{2x}}$$

$$\Leftrightarrow 2 = 3x$$

$$\Leftrightarrow x = \frac{2}{3} \qquad S : \left\{\frac{2}{3}\right\}$$

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(e) $\frac{2}{x-9}+1=0$ $\qquad$ CE : $x \neq 9$

$$\frac{2+x-9}{x-9}=0$$

$x-7=0$ $\qquad$ S : $\{7\}$

(f) $\frac{x+1}{x-1}-2=\frac{2x}{x-1}$ $\qquad$ CE : $x \neq 1$

$$\Leftrightarrow \frac{x+1-2(x-1)}{\cancel{x-1}}=\frac{2x}{\cancel{x-1}}$$

$\Leftrightarrow -x+3=2x$

$\Leftrightarrow 3=3x$

$\Leftrightarrow x=1 \rightarrow$ A.R. $\qquad$ S : $\emptyset$

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(g) $\frac{1}{x} - 2 + x = 0$ CE : $x \neq 0$

$$\Leftrightarrow \frac{1 - 2x + x^2}{x} = 0$$

$$\Leftrightarrow (x-1)^2 = 0$$

$$\Leftrightarrow x = 1$$

S : $\{1\}$

(h) $\frac{x}{5} - \frac{x+2}{x-2} = -\frac{4}{5}$ CE : $x \neq 2$

$$\Leftrightarrow \frac{x(x-2) - 5(x+2)}{5\cancel{(x-2)}} = \frac{-4(x-2)}{5\cancel{(x-2)}}$$

$$\Leftrightarrow x^2 - 2x - 5x - 10 = -4x + 8$$

$$\Leftrightarrow x^2 - 3x - 18 = 0$$

$$\overset{(1)}{\Leftrightarrow} (x+3)(x-6) = 0$$

$$\Leftrightarrow \begin{cases} x = -3 \\ x = 6 \end{cases}$$

S : $\{-3, 6\}$

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(i) $\dfrac{1}{x} - \dfrac{1}{10} = \dfrac{1}{x+5}$ $\qquad$ $\underline{CE}$ : $x \neq 0$ , $x \neq -5$

$$\Leftrightarrow \frac{10(x+5) - x(x+5)}{\cancel{10x(x+5)}} = \frac{10x}{\cancel{10x(x+5)}}$$

$$\Leftrightarrow \cancel{10x} + 50 - x^2 - 5x - \cancel{10x} = 0$$

$$\Leftrightarrow -x^2 - 5x + 50 = 0$$

n

$$\Leftrightarrow (-x+5)(x+10) = 0$$

$$\Leftrightarrow \begin{cases} -x+5=0 \\ x+10=0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x=5 \\ x=-10 \end{cases} \qquad S : \{-10, 5\}$$

(j) $\dfrac{9}{x^2+6x} - \dfrac{x-2}{2x+12} = \dfrac{1}{2x}$

$$\Leftrightarrow \frac{9}{x(x+6)} - \frac{x-2}{2(x+6)} = \frac{1}{2x} \qquad \underline{CE} \quad x \neq 0, x \neq -6$$

$$\Leftrightarrow \frac{18 - x(x-2)}{\cancel{2x(x+6)}} = \frac{(x+6)}{\cancel{2x(x+6)}}$$

$$\Leftrightarrow 18 - x^2 + 2x = x + 6$$

$$\Leftrightarrow x^2 - x - 12 = 0$$

n

$$\Leftrightarrow (x+3)(x-4) = 0$$

$$\Leftrightarrow \begin{cases} x+3=0 \\ x-4=0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x=-3 \\ x=4 \end{cases} \qquad S : \{-3, 4\}$$

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(k) $\frac{x^2}{x-2} - \frac{4x}{x+2} = \frac{8x}{x^2-4}$ $\quad$ $\underline{CE}$ : $x \neq \pm 2$

$$\Leftrightarrow \frac{x^2(x+2) - 4x(x-2)}{\cancel{(x-2)(x+2)}} = \frac{8x}{\cancel{x^2-4}}$$

$$\Leftrightarrow x^3 + 2x^2 - 4x^2 + \cancel{8x} = \cancel{8x}$$

$$\Leftrightarrow x^3 - 2x^2 = 0$$

$$\Leftrightarrow x^2(x-2) = 0$$

$$\Leftrightarrow \begin{cases} x^2 = 0 \\ x - 2 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 0 \\ x = 2 \ (A.R) \end{cases} \qquad S: \{0\}$$

(l) $\frac{1}{2} + \frac{x+1}{2x+2} = \frac{x}{3x+3}$ $\quad$ $\underline{CE}$ : $x \neq -1$

$$\Leftrightarrow \frac{3x+3+3(x+1)}{\cancel{6x+6}} = \frac{2x}{\cancel{6x+6}}$$

$$\Leftrightarrow 6x + 6 = 2x$$

$$\Leftrightarrow 4x = -6$$

$$\Leftrightarrow x = -\frac{3}{2} \qquad S: \{-\frac{3}{2}\}$$

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(m) $\frac{x}{x-1}+\frac{1}{x}-\frac{1}{x^2-x}=\frac{x}{x^2-x}$ CE : $x \neq 0$, $x \neq 1$

$\Leftrightarrow \frac{x^2+x-1-1}{x\cancel{(x-1)}}=\frac{x}{x\cancel{(x-1)}}$

$\Leftrightarrow x^2-2=0$

$\Leftrightarrow x=\pm\sqrt{2}$ S : $\{-\sqrt{2}, \sqrt{2}\}$

(n) $\frac{3x-1}{2x+8}-\frac{2x-3}{4(x+1)}=\frac{13}{40}$ CE : $x \neq -4$, $x \neq -1$

$\Leftrightarrow \frac{20(3x-1)(x+1)-10(2x-3)(x+4)}{40\cancel{(x+1)(x+4)}}=\frac{13(x+1)(x+4)}{40\cancel{(x+1)(x+4)}}$

$\Leftrightarrow 20(3x^2+2x-1)-10(2x^2+5x-12)=13(x^2+5x+4)$

$\Leftrightarrow 40x^2-10x+100=13x^2+65x+52$

$\Leftrightarrow 27x^2-75x+48=0$

$\Leftrightarrow 9x^2-25x+16=0$

R

$\Leftrightarrow (x-1)(9x-16)=0$

$\Leftrightarrow \begin{cases} x=1 \\ x=\frac{16}{9} \end{cases}$ S : $\{1, \frac{16}{9}\}$

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(o) $\frac{x-1}{x^2+3x}+\frac{2}{x}+\frac{9}{2x+6}=0$ $\underline{CE}$ : $x \neq -3$, $x \neq 0$

$\Leftrightarrow \frac{2(x-1)+4(x+3)+9x}{\cancel{2x(x+3)}}=0$

$\Leftrightarrow 2x-2+4x+12+9x=0$

$\Leftrightarrow 15x+10=0$

$\Leftrightarrow x=-\frac{2}{3}$ $S: \left\{-\frac{2}{3}\right\}$

(p) $\frac{2x}{x-3}-\frac{5}{x}=\frac{6x}{3x-9}+\frac{2}{3x}$ CE: $x \neq 0$, $x \neq 3$

$\Leftrightarrow \frac{\cancel{6x^2}-15(x-3)}{\cancel{3x(x-3)}}=\frac{\cancel{6x^2}+2(x-3)}{\cancel{3x(x-3)}}$

$\Leftrightarrow -15x+45=2x-6$

$\Leftrightarrow -17x=-51$

$\Leftrightarrow x=3 \rightarrow AR$ $S: \emptyset$

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