

**Semaine 3 :** Factoriser en utilisant les produits remarquables (une mise en évidence préalable peut parfois être nécessaire) :

$$1. 2a - 8a^3 + 8a^5 = 2a(1 - 4a^2 + 4a^4) \\ = 2a(1 - 2a^2)^2$$

$$2. 64x^5 + 16x^4 + x^3 = x^3(64x^2 + 16x + 1) \\ = x^3(8x + 1)^2$$

$$3. x^2 - 2 + \frac{1}{x^2} \quad a^3 - 4a^9 = a^3(1 - 4a^6) \\ = a^3(1 - 2a^3)(1 + 2a^3)$$

$$4. 100a^2 + 9b^6 - 30ab^3 \quad 25x^5y - 49xy^3 = xy(25x^4 - 49y^2) \\ = xy(5x^2 - 7y)(5x^2 + 7y)$$

$$5. 25x^4 + 16x^2y^4 - 40x^3y^2 = x^2(25x^2 + 16y^4 - 40xy^2) \\ = x^2(5x - 4y^2)^2$$

$$6. 4a^5 + 12a^3b^4 + 9ab^8 = a(4a^4 + 12a^2b^4 + 9b^8) \\ = a(2a^2 + 3b^4)^2$$

$$7. (2a + 1)^2 - (3 - a)^2 = [(2a + 1) - (3 - a)][(2a + 1) + (3 - a)] \\ = (3a - 2)(a + 4)$$

$$8. 9 + \frac{a^2b^2}{4} - 3ab = \left(3 - \frac{ab}{2}\right)^2$$

$$9. 25a^2 - (b - 2a)^2 = x^2 + 12x - 36 = -(x^2 - 12x + 36) \\ = -(x - 6)^2$$

Factoriser en utilisant les groupements :

$$\begin{aligned} 1. x^4 - 2x^3 + 4x - 8 &= (x^4 - 2x^3) + (4x - 8) \\ &= x^3(x - 2) + 4(x - 2) = (x - 2)(x^3 + 4) \end{aligned}$$

$$\begin{aligned} 2. 20a^3 + 5a^2 - 4a - 1 &= (20a^3 + 5a^2) - (4a + 1) \\ &= 5a^2(4a + 1) - (4a + 1) = (4a + 1)(5a^2 - 1) \end{aligned}$$

$$\begin{aligned} 3. 12ab - 8a + 3b - 2 &= (12ab - 8a) + (3b - 2) \\ &= 4a(3b - 2) + (3b - 2) = (4a + 1)(3b - 2) \end{aligned}$$

$$\begin{aligned} 4. a^3 - b^4 + a^2b - ab^3 &= (a^3 + a^2b) - (b^4 + ab^3) \\ &= a^2(a + b) - b^3(b + a) = (a + b)(a^2 - b^3) \end{aligned}$$

$$5. cy + y + c + 1 = (cy + y) + (c + 1)$$

$$= y(c + 1) + (c + 1) = (c + 1)(y + 1)$$

$$\begin{aligned} 6. 4x^2 + 2x - 9y^2 - 3y &= (4x^2 - 9y^2) + (2x - 3y) \\ &= (2x - 3y)(2x + 3y) + (2x - 3y) = (2x - 3y)(2x + 3y + 1) \end{aligned}$$

$$\begin{aligned} 7. c^2 + d - d^2 - c &= (c^2 - d^2) - (c - d) \\ &= (c - d)(c + d) - (c - d) = (c - d)(c + d - 1) \end{aligned}$$

$$\begin{aligned} 8. b^2y - b^2 + a^2y - a^2 &= (b^2y - b^2) + (a^2y - a^2) \\ &= b^2(y - 1) + a^2(y - 1) = (y - 1)(a^2 + b^2) \end{aligned}$$

$$\begin{aligned} 9. 5a^3 + a^2 - 20a - 4 &= (5a^3 + a^2) - (20a + 4) \\ &= a^2(5a + 1) - 4(5a + 1) \\ &= (5a + 1)(a^2 - 4) = (5a + 1)(a - 2)(a + 2) \end{aligned}$$

$$\begin{aligned} 10. a^4 - 2a^3 + a - 2 &= (a^4 - 2a^3) + (a - 2) \\ &= a^3(a - 2) + (a - 2) \\ &= (a - 2)(a^3 + 1) \end{aligned}$$