

## EXERCICES COMPLÉMENTAIRES : ÉQUATIONS DU SECOND DEGRÉ ET TRINÔME DU SECOND DEGRÉ

Résoudre les équations suivantes :

- |                            |                             |                            |
|----------------------------|-----------------------------|----------------------------|
| 1. $x^2 + 12x + 35 = 0$    | 11. $-4x^2 - 11x - 6 = 0$   | 21. $x^2 + 2x - 10 = 0$    |
| 2. $-48x^2 + 80x - 25 = 0$ | 12. $x^2 + 5x - 3 = 0$      | 22. $x^2 + 9x + 20 = 0$    |
| 3. $x^2 + 7x + 9 = 0$      | 13. $x^2 - x - 20 = 0$      | 23. $-27x^2 - 3x + 14 = 0$ |
| 4. $x^2 + 11x + 10 = 0$    | 14. $33x^2 - 113x + 10 = 0$ | 24. $x^2 + 2x - 7 = 0$     |
| 5. $-27x^2 + 30x - 7 = 0$  | 15. $x^2 + 6x + 3 = 0$      | 25. $x^2 + 5x = 0$         |
| 6. $-x^2 + 7x - 9 = 0$     | 16. $x^2 - 16x + 63 = 0$    | 26. $-x^2 - 5x - 4 = 0$    |
| 7. $x^2 - x - 42 = 0$      | 17. $11x^2 + 59x + 20 = 0$  | 27. $-x^2 + 9x = 0$        |
| 8. $-x^2 + 5x + 14 = 0$    | 18. $x^2 + 4x - 10 = 0$     | 28. $x^2 - 6x + 9 = 0$     |
| 9. $-x^2 + 9 = 0$          | 19. $x^2 + 4x - 21 = 0$     | 29. $-5x^2 + 49x - 36 = 0$ |
| 10. $x^2 + 15x + 54 = 0$   | 20. $-20x^2 + 9x + 18 = 0$  | 30. $x^2 + 6x - 4 = 0$     |

Factoriser les trinômes suivants :

- |                                  |                                 |
|----------------------------------|---------------------------------|
| 1. $Q(t) = t^2 - 7t$             | 17. $Q(y) = y^2 + 3y - 10$      |
| 2. $P(x) = -33x^2 - 10x + 8$     | 18. $S(z) = -12z^2 + 19z - 4$   |
| 3. $P(x) = -x^2 + 5x + 9$        | 19. $P(z) = z^2 + 2z - 2$       |
| 4. $P(y) = y^2 - 15y + 56$       | 20. $R(y) = y^2 + 10y$          |
| 5. $P(y) = 10y^2 - y - 24$       | 21. $R(z) = -80z^2 - 86z - 21$  |
| 6. $Q(y) = y^2 + 3y - 3$         | 22. $R(t) = -t^2 + 5t + 4$      |
| 7. $Q(z) = z^2 + 11z + 10$       | 23. $R(y) = 490y^2 + 140y + 10$ |
| 8. $R(y) = -33y^2 - 14y - 1$     | 24. $R(t) = t^2 + 2t$           |
| 9. $P(y) = y^2 + 2y + 6$         | 25. $Q(t) = 21t^2 + 34t + 9$    |
| 10. $Q(x) = x^2 - 14x + 48$      | 26. $Q(t) = -t^2 + 8t + 3$      |
| 11. $Q(t) = 16t^2 + 40t + 9$     | 27. $S(t) = t^2 + 15t + 50$     |
| 12. $S(z) = -z^2 + 5z + 4$       | 28. $Q(y) = 70y^2 + 73y + 9$    |
| 13. $S(y) = 405y^2 + 630y + 245$ | 29. $S(z) = -z^2 + 5z + 5$      |
| 14. $R(z) = z^2 + 2z - 15$       | 30. $S(z) = z^2 - z - 20$       |
| 15. $P(t) = 28t^2 + 13t - 6$     | 31. $S(y) = 10y^2 + 9y - 1$     |
| 16. $R(z) = -z^2 - 9$            | 32. $Q(z) = z^2 + 4z + 2$       |

Résoudre les inéquations suivantes :

1.  $\frac{8 - x^3}{4x^2 + 12x + 9} \leq 0$

2.  $\frac{x - 3}{x^2 - 4} - \frac{x}{x - 2} \geq \frac{x - 1}{x + 2}$

3.  $\frac{36x - 47}{-2x^2 + 7x + 9} > \frac{3x - 5}{x + 1} - \frac{-x + 2}{-2x + 9}$

4.  $\frac{2x - 1}{4 - x} < \frac{3 - 2x}{x + 3}$

5.  $\frac{2x^2 + x + 5}{x^2 - 3x + 3} < 1$

6.  $\frac{3x - 5}{-3x + 15} > \frac{-x + 3}{x + 1}$

7.  $(x - 1)(x - 2) < 1$

8.  $\frac{24 - 4x^2 - 20x}{4x^2 - 12x + 9} \leq -1$

9.  $2 \geq \frac{x^2 - 2x}{x^2 + 2x} + \frac{x^2 + 2x}{x^2 - 2x}$

## Second degré - Exercices complémentaires : Solutions

Résoudre les équations suivantes :

1.  $x^2 + 12x + 35 = 0$

$$\Delta = 144 - 140 = 4 \quad x_{1,2} = \frac{-12 \pm 2}{2} < \begin{matrix} -5 \\ -7 \end{matrix}$$

$$S : \{-7, -5\}$$

2.  $-48x^2 + 80x - 25 = 0$

$$\Delta = 6400 - 4800 = 1600 \quad x_{1,2} = \frac{-80 \pm 40}{-96} < \begin{matrix} +\frac{5}{12} \\ \frac{5}{4} \end{matrix}$$

$$S : \left\{ \frac{5}{12}, \frac{5}{4} \right\}$$

3.  $x^2 + 7x + 9 = 0$

$$\Delta = 49 - 36 = 7 \quad x_{1,2} = \frac{-7 \pm \sqrt{7}}{2}$$

$$S : \left\{ -\frac{7+\sqrt{7}}{2}, -\frac{7-\sqrt{7}}{2} \right\}$$

4.  $x^2 + 11x + 10 = 0$

$$\Delta = 121 - 40 = 81 \quad x_{1,2} = \frac{-11 \pm 9}{2} < \begin{matrix} -1 \\ -10 \end{matrix}$$

$$S : \{-10, -1\}$$

$$5. -27x^2 + 30x - 7 = 0$$

$$\Delta = 900 - 756 = 144$$

$$x_{1,2} = \frac{-30 \pm 12}{-54} < \begin{matrix} \frac{1}{3} \\ \frac{2}{9} \end{matrix}$$

$$S: \left\{ \frac{1}{3}, \frac{2}{9} \right\}$$

$$6. -x^2 + 7x - 9 = 0$$

$$\Delta = 49 - 36 = 7$$

$$x_{1,2} = \frac{-7 \pm \sqrt{7}}{-2}$$

$$S: \left\{ \frac{7-\sqrt{7}}{2}, \frac{7+\sqrt{7}}{2} \right\}$$

$$7. x^2 - x - 42 = 0$$

$$\Delta = 1 + 168 = 169$$

$$x_{1,2} = \frac{1 \pm 13}{2} < \begin{matrix} 7 \\ -6 \end{matrix}$$

$$S: \{-6, 7\}$$

$$8. -x^2 + 5x + 14 = 0$$

$$\Delta = 25 + 56 = 81$$

$$x_{1,2} = \frac{-5 \pm 9}{-2} < \begin{matrix} -2 \\ 7 \end{matrix}$$

$$S: \{-2, 7\}$$

$$9. -x^2 + 9 = 0$$

$$\Delta = 36$$

$$x_{1,2} = \frac{0 \pm 6}{-2} < \begin{matrix} -3 \\ 3 \end{matrix}$$

$$S: \{-3, 3\}$$

$$10. x^2 + 15x + 54 = 0$$

$$\Delta = 225 - 216 = 9$$

$$x_{1,2} = \frac{-15 \pm 3}{2} \begin{matrix} -6 \\ -9 \end{matrix}$$

$$S: \{-9, -6\}$$

$$11. -4x^2 - 11x - 6 = 0$$

$$\Delta = 121 - 96 = 25$$

$$x_{1,2} = \frac{11 \pm 5}{-8} \begin{matrix} -2 \\ \frac{3}{4} \end{matrix}$$

$$S: \{-2, \frac{3}{4}\}$$

$$12. x^2 + 5x - 3 = 0$$

$$\Delta = 25 + 12 = 37$$

$$x_{1,2} = \frac{-5 \pm \sqrt{37}}{2}$$

$$S: \left\{ \frac{-5 - \sqrt{37}}{2}, \frac{-5 + \sqrt{37}}{2} \right\}$$

$$13. x^2 - x - 20 = 0$$

$$\Delta = 1 + 80 = 81$$

$$x_{1,2} = \frac{1 \pm 9}{2} \begin{matrix} 5 \\ -4 \end{matrix}$$

$$S: \{-4, 5\}$$

$$14. 33x^2 - 113x + 10 = 0$$

$$\Delta = 12769 - 1320 = 11449$$

$$x_{1,2} = \frac{113 \pm 107}{66} \begin{matrix} \frac{10}{3} \\ \frac{1}{11} \end{matrix}$$

$$S: \left\{ \frac{1}{11}, \frac{10}{3} \right\}$$

$$15. x^2 + 6x + 3 = 0$$

$$\Delta = 36 - 12 = 24 \quad x_{1,2} = \frac{-6 \pm 2\sqrt{6}}{2} = \frac{-3 \pm \sqrt{6}}{1}$$

$$S: \{-3 - \sqrt{6}, -3 + \sqrt{6}\}$$

$$16. x^2 - 16x + 63 = 0$$

$$\Delta = 256 - 252 = 4 \quad x_{1,2} = \frac{16 \pm 2}{2} \in \begin{matrix} 9 \\ 7 \end{matrix}$$

$$S: \{7, 9\}$$

$$17. 11x^2 + 59x + 20 = 0$$

$$\Delta = 3481 - 880 = 2601 \quad x_{1,2} = \frac{-59 \pm 51}{22} \in \begin{matrix} -\frac{4}{11} \\ 5 \end{matrix}$$

$$S: \{-\frac{4}{11}, 5\}$$

$$18. x^2 + 4x - 10 = 0$$

$$\Delta = 16 + 40 = 56 \quad x_{1,2} = \frac{-4 \pm 2\sqrt{14}}{2} = \frac{-2 \pm \sqrt{14}}{1}$$

$$S: \{-2 - \sqrt{14}, -2 + \sqrt{14}\}$$

$$19. x^2 + 4x - 21 = 0$$

$$\Delta = 16 + 84 = 100 \quad x_{1,2} = \frac{-4 \pm 10}{2} \in \begin{matrix} 3 \\ -7 \end{matrix}$$

$$S: \{-7, 3\}$$

$$20. -20x^2 + 9x + 18 = 0$$

$$\Delta = 81 + 1440 = 1521$$

$$x_{1,2} = \frac{-9 \pm 39}{-40} < \begin{matrix} -\frac{3}{4} \\ \frac{6}{5} \end{matrix}$$

$$S: \left\{ -\frac{3}{4}, \frac{6}{5} \right\}$$

$$21. x^2 + 2x - 10 = 0$$

$$\Delta = 4 + 40 = 44$$

$$x_{1,2} = \frac{-2 \pm 2\sqrt{11}}{2} = \frac{-1(1 \pm \sqrt{11})}{1}$$

$$S: \left\{ -1 - \sqrt{11}, -1 + \sqrt{11} \right\}$$

$$22. x^2 + 9x + 20 = 0$$

$$\Delta = 81 - 80 = 1$$

$$x_{1,2} = \frac{-9 \pm 1}{2} < \begin{matrix} -4 \\ -5 \end{matrix}$$

$$S: \left\{ -5, -4 \right\}$$

$$23. -27x^2 - 3x + 14 = 0$$

$$\Delta = 9 + 1512 = 1521$$

$$x_{1,2} = \frac{3 \pm 39}{54} < \begin{matrix} \frac{7}{9} \\ -\frac{2}{3} \end{matrix}$$

$$S: \left\{ -\frac{2}{3}, \frac{7}{9} \right\}$$

$$24. x^2 + 2x - 7 = 0$$

$$\Delta = 4 + 28 = 32$$

$$x_{1,2} = \frac{-2 \pm 4\sqrt{2}}{2} = \frac{-1(1 \pm 2\sqrt{2})}{1}$$

$$S: \left\{ -1 - 2\sqrt{2}, -1 + 2\sqrt{2} \right\}$$

$$25. x^2 + 5x = 0$$

$$\Delta = 25$$

$$x_{1,2} = \frac{-5 \pm 5}{2} \begin{matrix} < 0 \\ -5 \end{matrix}$$

$$S: \{-5, 0\}$$

$$26. -x^2 - 5x - 4 = 0$$

$$\Delta = 25 - 16 = 9 \quad x_{1,2} = \frac{5 \pm 3}{-2} \begin{matrix} < -4 \\ -1 \end{matrix}$$

$$S: \{-4, -1\}$$

$$27. -x^2 + 9x = 0$$

$$\Delta = 81$$

$$x_{1,2} = \frac{-9 \pm 9}{-2} \begin{matrix} < 0 \\ 9 \end{matrix}$$

$$S: \{0, 9\}$$

$$28. x^2 - 6x + 9 = 0$$

$$\Delta = 36 - 36 = 0$$

$$x_{1,2} = \frac{6}{2} = 3$$

$$S: \{3\}$$

$$29. -5x^2 + 49x - 36 = 0$$

$$\Delta = 2401 - 720 = 1681$$

$$x_{1,2} = \frac{-49 \pm 41}{-10} \begin{matrix} < \frac{1}{5} \\ 9 \end{matrix}$$

$$S: \left\{\frac{4}{5}, 9\right\}$$

$$30. x^2 + 6x - 4 = 0$$

$$\Delta = 36 + 16 = 52$$

$$x_{1,2} = \frac{-6 \pm 2\sqrt{13}}{2} = \frac{-2(3 \pm \sqrt{13})}{2}$$

$$S: \left\{ -3 - \sqrt{13}, -3 + \sqrt{13} \right\}$$

Factoriser les trinômes suivants :

$$1. Q(t) = t^2 - 7t$$

\* PEE = mise en évidence

$$Q(t) = t(t-7)$$

PEE

$$2. P(x) = -33x^2 - 10x + 8$$

$$\Delta = 100 + 1056 = 1156$$

$$x_{1,2} = \frac{10 \pm 34}{-66} < \begin{matrix} -\frac{2}{3} \\ \frac{4}{11} \end{matrix}$$

$$P(x) = -33 \left( x + \frac{2}{3} \right) \left( x - \frac{4}{11} \right) = - (3x + 2) (11x - 4)$$

$$3. P(x) = -x^2 + 5x + 9$$

$$\Delta = 25 + 36 = 61$$

$$x_{1,2} = \frac{-5 \pm \sqrt{61}}{2}$$

$$\begin{aligned} P(x) &= - \left( x - \frac{-5 + \sqrt{61}}{2} \right) \left( x - \frac{-5 - \sqrt{61}}{2} \right) \\ &= - \left( x + \frac{5 - \sqrt{61}}{2} \right) \left( x + \frac{5 + \sqrt{61}}{2} \right) \end{aligned}$$

$$4. P(y) = y^2 - 15y + 56$$

$$\Delta = 225 - 224$$

$$y_{1,2} = \frac{15 \pm 1}{2} < \begin{matrix} 8 \\ 7 \end{matrix}$$

$$P(y) = (y - 8)(y - 7)$$

$$5. P(y) = 10y^2 - y - 24$$

$$\Delta = 1 + 960 = 961 \quad y_{1,2} = \frac{1 \pm 31}{20} < \frac{8}{5} \quad -\frac{3}{2}$$

$$P(y) = 10 \left( y - \frac{8}{5} \right) \left( y + \frac{3}{2} \right) = (5y - 8)(2y + 3)$$

$$6. Q(y) = y^2 + 3y - 3$$

$$\Delta = 9 + 12 = 21 \quad y_{1,2} = \frac{-3 \pm \sqrt{21}}{2}$$

$$Q(y) = \left( y - \frac{3 + \sqrt{21}}{2} \right) \left( y - \frac{3 - \sqrt{21}}{2} \right)$$

$$7. Q(z) = z^2 + 11z + 10$$

$$\Delta = 121 - 40 = 81 \quad z_{1,2} = \frac{-11 \pm 9}{2} < -1 \quad -10$$

$$Q(z) = (z + 1)(z + 10)$$

$$8. R(y) = -33y^2 - 14y - 1$$

$$\Delta = 196 - 132 = 64 \quad y_{1,2} = \frac{-14 \pm 8}{-66} < -\frac{1}{3} \quad -\frac{1}{11}$$

$$R(y) = -33 \left( y + \frac{1}{3} \right) \left( y + \frac{1}{11} \right) = -(3y + 1)(11y + 1)$$

$$9. P(y) = y^2 + 2y + 6$$

$$\Delta = 4 - 24 = -20$$

$P(y)$  n'est pas factorisable

$$10. Q(x) = x^2 - 14x + 48$$

$$\Delta = 196 - 192 = 4$$

$$x_{1,2} = \frac{+14 \pm 2}{2} < \frac{8}{6}$$

$$Q(x) = (x - 8)(x - 6)$$

$$11. Q(t) = 16t^2 + 40t + 9$$

$$\Delta = 1600 - 576 = 1024$$

$$t_{1,2} = \frac{-40 \pm 32}{32} < \begin{matrix} \frac{1}{4} \\ -\frac{9}{4} \end{matrix}$$

$$Q(t) = 16 \left( t + \frac{1}{4} \right) \left( t + \frac{9}{4} \right) = (4t + 1)(4t + 9)$$

$$12. S(z) = -z^2 + 5z + 4$$

$$\Delta = 25 + 16 = 41$$

$$z_{1,2} = \frac{-5 \pm \sqrt{41}}{-2} < \begin{matrix} \frac{5 - \sqrt{41}}{2} \\ \frac{5 + \sqrt{41}}{2} \end{matrix}$$

$$S(z) = - \left( z - \frac{5 - \sqrt{41}}{2} \right) \left( z - \frac{5 + \sqrt{41}}{2} \right)$$

$$13. S(y) = 405y^2 + 630y + 245$$

$$\Delta = 396900 - 396900 = 0$$

$$y_{1,2} = \frac{-630}{810} = -\frac{7}{9}$$

$$S(y) = \underbrace{405}_{5 \cdot 81} \left( y + \frac{7}{9} \right)^2 = 5 \left( 9y + 7 \right)^2$$

$$14. R(z) = z^2 + 2z - 15$$

$$\Delta = 4 + 60 = 64$$

$$z_{1,2} = \frac{-2 \pm 8}{2} < \begin{matrix} 3 \\ -5 \end{matrix}$$

$$R(z) = (z - 3)(z + 5)$$

$$15. P(t) = 28t^2 + 13t - 6$$

$$\Delta = 169 + 672 = 841 \quad t_{1,2} = \frac{-13 \pm 29}{56} < \begin{matrix} \frac{2}{7} \\ -\frac{3}{4} \end{matrix}$$

$$P(t) = 28 \left( t - \frac{2}{7} \right) \left( t + \frac{3}{4} \right) = (7t - 2)(4t + 3)$$

$$16. R(z) = -z^2 - 9$$

$$R(z) = -(z^2 + 9) \quad (\Delta < 0)$$

$$17. Q(y) = y^2 + 3y - 10$$

$$\Delta = 9 + 40 = 49 \quad y_{1,2} = \frac{-3 \pm 7}{2} < \begin{matrix} 2 \\ -5 \end{matrix}$$

$$Q(y) = (y - 2)(y + 5)$$

$$18. S(z) = -12z^2 + 19z - 4$$

$$\Delta = 361 - 192 = 169 \quad z_{1,2} = \frac{-19 \pm 13}{-24} < \begin{matrix} \frac{1}{4} \\ \frac{4}{3} \end{matrix}$$

$$S(z) = -12 \left( z - \frac{1}{4} \right) \left( z - \frac{4}{3} \right) \\ = -(4z - 1)(3z - 4)$$

$$19. P(z) = z^2 + 2z - 2$$

$$\Delta = 4 + 8 = 12 \quad z_{1,2} = \frac{-2 \pm 2\sqrt{3}}{2} < \begin{matrix} -1 + \sqrt{3} \\ -1 - \sqrt{3} \end{matrix}$$

$$P(z) = (z + 1 - \sqrt{3})(z + 1 + \sqrt{3})$$

$$20. R(y) = y^2 + 10y$$

$$R(y) = y(y + 10)$$

$$21. R(z) = -80z^2 - 86z - 21$$

$$D = 7396 - 6720 = 676 \quad z_{1,2} = \frac{86 \pm 26}{-160} < \begin{matrix} -\frac{7}{10} \\ -\frac{3}{8} \end{matrix}$$

$$R(z) = -80 \left( z + \frac{7}{10} \right) \left( z + \frac{3}{8} \right) \\ = -(10z + 7)(8z + 3)$$

$$22. R(t) = -t^2 + 5t + 4$$

$$D = 25 + 16 = 41 \quad t_{1,2} = \frac{-5 \pm \sqrt{41}}{-2} < \begin{matrix} \frac{5 - \sqrt{41}}{2} \\ \frac{5 + \sqrt{41}}{2} \end{matrix}$$

$$R(t) = - \left( t - \frac{5 - \sqrt{41}}{2} \right) \left( t - \frac{5 + \sqrt{41}}{2} \right)$$

$$23. R(y) = 490y^2 + 140y + 10$$

$$D = 19600 - 19600 = 0 \quad y_{1,2} = \frac{-140}{980} = -\frac{1}{7}$$

$$R(y) = \underbrace{490}_{49 \cdot 10} \left( y + \frac{1}{7} \right)^2 = 10 (7y + 1)^2$$

$$24. R(t) = t^2 + 2t$$

$$R(t) = t(t + 2)$$

$$25. Q(t) = 21t^2 + 34t + 9$$

$$\Delta = 1156 - 756 = 400$$

$$t_{1,2} = \frac{-34 \pm 20}{42} < \begin{matrix} -\frac{9}{7} \\ -\frac{1}{3} \end{matrix}$$

$$Q(t) = 21 \left( t + \frac{9}{7} \right) \left( t + \frac{1}{3} \right) \\ = (7t + 9)(3t + 1)$$

$$26. Q(t) = -t^2 + 8t + 3$$

$$\Delta = 64 + 12 = 76$$

$$t_{1,2} = \frac{-8 \pm 2\sqrt{19}}{-2} = 4 \pm \sqrt{19}$$

$$Q(t) = - (t - 4 - \sqrt{19})(t - 4 + \sqrt{19})$$

$$27. S(t) = t^2 + 15t + 50$$

$$\Delta = 225 - 200 = 25$$

$$t_{1,2} = \frac{-15 \pm 5}{2} < \begin{matrix} -5 \\ -10 \end{matrix}$$

$$S(t) = (t + 10)(t + 5)$$

$$28. Q(y) = 70y^2 + 73y + 9$$

$$\Delta = 5329 - 2520 = 2809$$

$$y_{1,2} = \frac{-73 \pm 53}{140} < \begin{matrix} -\frac{3}{10} \\ -\frac{1}{7} \end{matrix}$$

$$Q(y) = 70 \left( y + \frac{9}{10} \right) \left( y + \frac{1}{7} \right) = (6y + 9)(7y + 1)$$

$$29. S(z) = -z^2 + 5z + 5$$

$$\Delta = 25 + 20 = 45$$

$$z_{1,2} = \frac{-5 \pm 3\sqrt{5}}{-2} = \frac{5 \pm 3\sqrt{5}}{2}$$

$$S(z) = - \left( z - \frac{5 - 3\sqrt{5}}{2} \right) \left( z - \frac{5 + 3\sqrt{5}}{2} \right)$$

$$30. S(z) = z^2 - z - 20$$

$$\Delta = 1 + 80 = 81$$

$$z_{1,2} = \frac{1 \pm 9}{2} \begin{matrix} 5 \\ -4 \end{matrix}$$

$$S(z) = (z - 5)(z + 4)$$

$$31. S(y) = 10y^2 + 9y - 1$$

$$\Delta = 81 + 40 = 121$$

$$y_{1,2} = \frac{-9 \pm 11}{20} \begin{matrix} \frac{1}{10} \\ -1 \end{matrix}$$

$$S(y) = 10 \left( y - \frac{1}{10} \right) (y + 1) \\ = (10y - 1)(y + 1)$$

$$32. Q(z) = z^2 + 4z + 2$$

$$\Delta = 16 - 8 = 8$$

$$z_{1,2} = \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$$

$$Q(z) = (z + 2 - \sqrt{2})(z + 2 + \sqrt{2})$$

Résoudre les inéquations suivantes :

1.  $\frac{8 - x^3}{4x^2 + 12x + 9} \leq 0$

zéros: N :  $x^3 - 8 = 0 \Rightarrow x = 2$

D :  $4x^2 + 12x + 9 = 0$

$\Delta = 144 - 144 = 0$

$x_1 = -\frac{12}{8} = -\frac{3}{2}$

| $x$              | $-\frac{3}{2}$ | $2$ |     |
|------------------|----------------|-----|-----|
| $8 - x^3$ *      | +              | +   | 0 - |
| $4x^2 + 12x + 9$ | +              | +   | +   |
| $I_n$            | +              | +   | 0 - |



$S : [2, +\infty$

⊗ Le comportement d'une expression du 3<sup>e</sup> degré est le même que celle du 1<sup>er</sup> degré (si celle-ci ne s'annule qu'une fois)

$$2. \frac{x-3}{x^2-4} - \frac{x}{x-2} \geq \frac{x-1}{x+2}$$

$$\Leftrightarrow \frac{x-3}{\underbrace{x^2-4}_{(x-2)(x+2)}} - \frac{x}{x-2} - \frac{x-1}{x+2} \geq 0$$

$$\Leftrightarrow \frac{x-3 - x(x+2) - (x-1)(x-2)}{(x-2)(x+2)} \geq 0$$

$$\Leftrightarrow \frac{x-3 - (x^2+2x) - (x^2-2x-x+2)}{(x-2)(x+2)} \geq 0$$

$$\Leftrightarrow \frac{-2x^2+2x-5}{(x-2)(x+2)} \geq 0$$

3. Lösung: N:  $-2x^2+2x-5=0$

$$\Delta = 4 - 40 = -36 < 0 \quad \checkmark$$

D:  $x = -2, x = 2$

| $x$          |  | $-2$ |            | $2$ |            |
|--------------|--|------|------------|-----|------------|
|              |  | $ $  |            | $ $ |            |
| $-2x^2+2x-5$ |  | -    |            | -   | -          |
| $x-2$        |  | -    |            | 0   | +          |
| $x+2$        |  | -    | 0          | +   | +          |
| $I_n$        |  | -    | $\nexists$ | +   | $\nexists$ |
|              |  |      | $x$        |     | $x$        |

$$S: ]-2, 2[$$

$$3. \frac{36x-47}{-2x^2+7x+9} > \frac{3x-5}{x+1} - \frac{-x+2}{-2x+9}$$

$$-2x^2+7x+9 \stackrel{\Delta}{=} (-2x+9)(x+1)$$

$$\Leftrightarrow \frac{36x-47}{(-2x+9)(x+1)} - \frac{3x-5}{x+1} + \frac{-x+2}{-2x+9} > 0$$

$$\Leftrightarrow \frac{36x-47 - (3x-5)(-2x+9) + (-x+2)(x+1)}{(-2x+9)(x+1)} > 0$$

$$\Leftrightarrow \frac{36x-47 - (-6x^2+27x+45) + (-x^2-x+2x+2)}{(-2x+9)(x+1)} > 0$$

$$\Leftrightarrow \frac{5x^2}{(-2x+9)(x+1)} > 0$$

Zeilen: N :  $x=0$

D :  $x=-1, x=\frac{9}{2}$

| $x$     | -1 | 0 | $\frac{9}{2}$ |
|---------|----|---|---------------|
| $5x^2$  | +  | 0 | +             |
| $-2x+9$ | +  | + | 0             |
| $x+1$   | -  | + | +             |
| $I_n$   | -  | + | -             |

x ————— x ————— x

S:  $] -1, 0[ \cup ] 0, \frac{9}{2}[$

$$(d) \frac{2x-1}{4-x} < \frac{3-2x}{x+3}$$

$$\frac{2x^2 + 5x - 3 - (12 + 2x^2 - 11x)}{(4-x)(x+3)} < 0$$

$$\frac{16x - 15}{(4-x)(x+3)} < 0$$

| $x$      |  | $-3$ | $\frac{15}{16}$ | $4$ |   |
|----------|--|------|-----------------|-----|---|
| $16x-15$ |  | -    | -               | +   | + |
| $4-x$    |  | +    | +               | +   | - |
| $x+3$    |  | -    | +               | -   | + |
| $I_n$    |  | +    | -               | +   | - |

$$S: ]-3, \frac{15}{16}[ \cup ]4, +\infty[$$

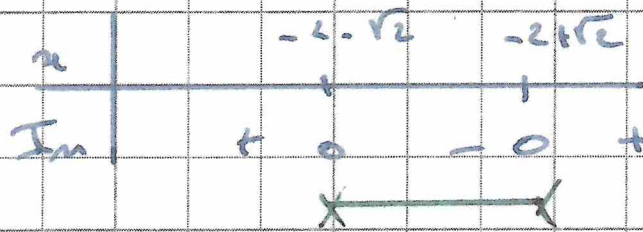
$$(e) \frac{2x^2 + x + 5}{x^2 - 3x + 3} < 1$$

$$\frac{2x^2 + x + 5 - x^2 + 3x - 3}{x^2 - 3x + 3} < 0$$

$$\frac{x^2 + 4x + 2}{x^2 - 3x + 3} < 0$$

Zeige: N :  $\Delta = 16 - 8 = 8 \quad x_{1,2} = \frac{-4 \pm 2\sqrt{2}}{2}$   
 $= -2 \pm \sqrt{2}$

D:  $\Delta = 9 - 12 < 0 \rightarrow D > 0$



S:  $] -2 - \sqrt{2}, -2 + \sqrt{2} [$

$$(i) \frac{3x-5}{-3x+15} > \frac{-x+3}{x+1}$$

$$\Leftrightarrow \frac{(3x-5)(x+1) - (-x+3)(-3x+15)}{(-3x+15)(x+1)} > 0$$

$$\Leftrightarrow \frac{3x^2 - 2x - 5 - (3x^2 - 24x + 45)}{(-3x+15)(x+1)} > 0$$

$$\Leftrightarrow \frac{22x - 50}{(-3x+15)(x+1)} > 0$$

| x     | -1 | $\frac{25}{11}$ | 5 |   |
|-------|----|-----------------|---|---|
| 2     | -  | -               | + | + |
| 0     | -  | +               | + | - |
| $I_n$ | +  | -               | + | - |
|       | x  | x               | x | x |

$$S: -\infty, -1 \cup \left[ \frac{25}{11}, 5 \right)$$

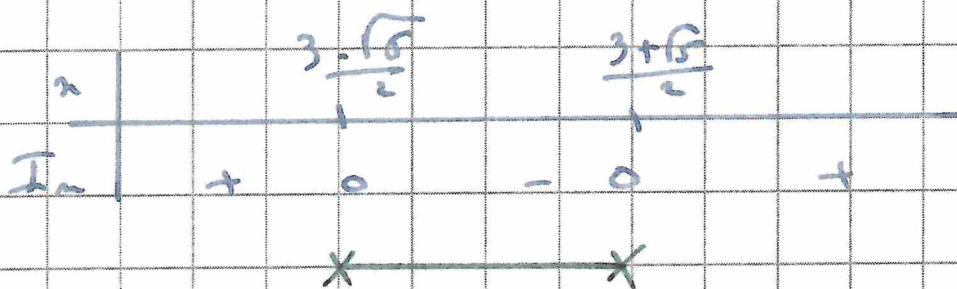
$$(k) (x-1)(x-2) < 1$$

$$\Leftrightarrow x^2 - 3x + 2 - 1 < 0$$

$$\Leftrightarrow x^2 - 3x + 1 < 0$$

discriminant:  $\Delta = 9 - 4 = 5$

$$x_{1,2} = \frac{3 \pm \sqrt{5}}{2}$$

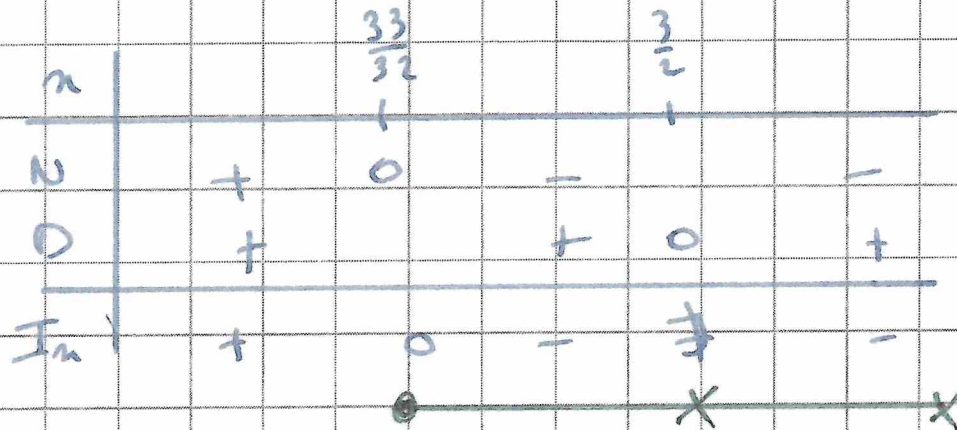


$$S: \left] \frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2} \right[$$

$$(o) \frac{24 - 4x^2 - 20x}{4x^2 - 12x + 9} \leq -1$$

$$\Leftrightarrow \frac{-4x^2 - 20x + 24 + (4x^2 - 12x + 9)}{(2x - 3)^2} \leq 0$$

$$\Leftrightarrow \frac{-32x + 33}{(2x - 3)^2} \leq 0$$



$$S : \left[ \frac{33}{32}, \frac{3}{2} \right[ \cup \left] \frac{3}{2}, +\infty \right[$$

$$(p) \quad 2 \geq \frac{x^2 - 2x}{x^2 + 2x} + \frac{x^2 + 2x}{x^2 - 2x}$$

$$\Leftrightarrow 2 \geq \frac{(x^2 - 2x)}{x(x+2)} + \frac{(x^2 + 2x)}{x(x-2)}$$

$$\Leftrightarrow \frac{2x(x-2)(x+2) - (x^2 - 2x)(x-2) - (x^2 + 2x)(x+2)}{x(x-2)(x+2)} \geq 0$$

$$\Leftrightarrow \frac{2x(x^2 - 4) - (x^3 - 2x^2 - 2x^2 + 4x) - (x^3 + 2x^2 + 2x^2 + 4x)}{D} \geq 0$$

$$\Leftrightarrow \frac{2x^3 - 8x - x^3 + 4x^2 - 4x - x^3 - 4x^2 - 4x}{D} \geq 0$$

$$\Leftrightarrow \frac{-16x}{x(x-2)(x+2)} \geq 0 \quad \Delta \quad \text{L.E.: } x \neq 0$$

| x         | -2 | 0 | 2 |
|-----------|----|---|---|
| -16       | -  | - | - |
| $x^2 - 4$ | +  | - | + |
| $I_n$     | -  | + | - |
|           | X  | X | X |

$$S: ]-2, 0[ \cup ]0, 2[$$