

Exercices complémentaires : Fractions algébriques, équations et inéquations réductibles au premier degré : Solutions

1. Simplifier après avoir donné les conditions d'existence :

$$(a) \frac{x-2}{2x} + \frac{1}{x^2} - \frac{1}{2}$$

CE : $x \neq 0$

$$= \frac{x(x-2) + 2 - x^2}{2x^2}$$

$$= \frac{x^2 - 2x + 2 - x^2}{2x^2}$$

D

$$= \frac{2(-x+1)}{2x^2}$$

$$= \frac{1-x}{x^2}$$

$$(b) \frac{x-2y}{x} + \frac{2x-y}{y}$$

CE : $x \neq 0, y \neq 0$

$$= \frac{y(x-2y) + x(2x-y)}{xy}$$

$$= \frac{xy - 2y^2 + 2x^2 - xy}{xy}$$

$$= \frac{2(x^2 - y^2)}{xy}$$

$$(c) \frac{1-x}{x} - \frac{2x^2-1}{2x^2} \quad \underline{\text{CE}} : n \neq 0$$

$$= \frac{2n(n-x) - (2n^2-1)}{2n^2}$$

$$= \frac{2n - 2n^2 - 2n^2 + 1}{2n^2}$$

$$= \frac{-4n^2 + 2n - 1}{2n^2}$$

$$(d) \frac{4x^2}{3x^2-3x} + \frac{5x}{x^2-1} - 1 = \frac{4x^2}{3x(x-1)} + \frac{5x}{(x-1)(x+1)} - 1$$

$$\underline{\text{CE}} : n \neq 0, n \neq 1, n \neq -1$$

$$= \frac{4n^2(n+1) + 15n^2 - 3n(n-1)(n+1)}{3n(n-1)(n+1)}$$

$$= \frac{4n^3 + 4n^2 + 15n^2 - 3n(n^2-1)}{3n(n-1)(n+1)}$$

$$= \frac{4n^3 + 13n^2 - 3n^3 + 3n}{3n(n-1)(n+1)}$$

$$= \frac{x(n^2 + 19n + 3)}{3x(n-1)(n+1)}$$

$$= \frac{x^2 + 19n + 3}{3(n-1)(n+1)}$$

$$(e) \frac{2a}{a-1} \textcircled{C} \frac{a^2+2a}{a^2-1} = \frac{2a}{a-1} - \frac{a^2+2a}{(a-1)(a+1)}$$

C.E.: $a \neq 1, a \neq -1$

$$\begin{aligned} &= \frac{2a(a+1) \textcircled{-} (a^2+2a)}{(a-1)(a+1)} \\ &= \frac{2a^2 + 2a - a^2 - 2a}{D} \\ &= \frac{a^2}{(a-1)(a+1)} \end{aligned}$$

$$(f) \frac{1}{x-2} - \frac{1}{x+2} + \frac{2x}{x^2-4} = \frac{1}{n-2} \textcircled{-} \frac{1}{n+2} + \frac{2x}{(n-2)(n+2)}$$

C.E.: $n \neq 2, n \neq -2$

$$\begin{aligned} &= \frac{x+2 \textcircled{-} (n-2) + 2n}{(n-2)(n+2)} \\ &= \frac{n+2 - n+2 + 2n}{D} \\ &= \frac{4+2n}{D} \\ &= \frac{2(2+n)}{(n-2)(n+2)} \\ &= \frac{2}{n-2} \end{aligned}$$

$$(g) \frac{2a}{a^2-1} - \frac{1}{a+1} - \frac{1}{a-1} = \frac{2a}{(a-1)(a+1)} - \frac{1}{a+1} - \frac{1}{a-1}$$

CE : $a \neq 1, a \neq -1$

$$\begin{aligned} &= \frac{2a - (a-1) - (a+1)}{(a-1)(a+1)} \\ &= \frac{2a - a + 1 - a - 1}{D} \\ &= 0 \end{aligned}$$

$$(h) \frac{3x}{x^2-xy} - \frac{2y}{xy+y^2} = \frac{3x}{x(x-y)} - \frac{2y}{y(x+y)}$$

CE : $x \neq 0, y \neq 0, x+y, x=-y$

$$\begin{aligned} &= \frac{3}{x-y} - \frac{2}{x+y} \\ &= \frac{3(x+y) - 2(x-y)}{(x-y)(x+y)} \\ &= \frac{3x+3y - 2x+2y}{D} \\ &= \frac{x+5y}{(x-y)(x+y)} \end{aligned}$$

$$(i) \frac{5}{2a-4b} - \frac{a-2b}{a^2-4b^2} = \frac{5}{2(a-2b)} - \frac{a-2b}{(a-2b)(a+2b)}$$

C6 $a \neq 2b$, $a \neq -2b$

$$\begin{aligned} &= \frac{5}{2(a-2b)} - \frac{1}{a+2b} \\ &= \frac{5(a+2b) - 2(a-2b)}{2(a-2b)(a+2b)} \\ &= \frac{5a+10b - 2a+4b}{2(a-2b)(a+2b)} \\ &= \frac{3a+14b}{2(a-2b)(a+2b)} \end{aligned}$$

$$(ii) \frac{3x+3y}{x^2-y^2} + \frac{7x-7y}{x^2-2xy+y^2} = \frac{3(x+y)}{(x-y)(x+y)} + \frac{7(x-y)}{(x-y)^2}$$

C6: $x \neq y$, $x \neq -y$

$$\begin{aligned} &= \frac{3}{x-y} + \frac{7}{x-y} \\ &= \frac{10}{x-y} \end{aligned}$$

2. Résoudre dans \mathbb{R} les équations suivantes :

F = factorisation

$$(a) x(2x-1)(3x+7) = 0$$

$$\Leftrightarrow \begin{cases} x=0 \\ 2x-1=0 \\ 3x+7=0 \end{cases} \Leftrightarrow \begin{cases} x=0 \\ x=\frac{1}{2} \\ x=-\frac{7}{3} \end{cases}$$

$$S: \left\{-\frac{7}{3}, 0, \frac{1}{2}\right\}$$

$$(b) x^2 = 64$$

$$\Leftrightarrow x^2 - 64 = 0 \stackrel{F}{\Leftrightarrow} (x-8)(x+8) = 0$$

$$\Leftrightarrow \begin{cases} x-8=0 \\ x+8=0 \end{cases} \Leftrightarrow \begin{cases} x=8 \\ x=-8 \end{cases}$$

$$S: \{-8, 8\}$$

$$(c) 9x^2 + 16 = -24x$$

$$\Leftrightarrow 9x^2 + 24x + 16 = 0 \stackrel{F}{\Leftrightarrow} (3x+4)^2 = 0$$

$$\Leftrightarrow 3x+4 = 0 \Leftrightarrow x = -\frac{4}{3}$$

$$S: \left\{-\frac{4}{3}\right\}$$

$$\textcircled{*} \quad a=0 \Leftrightarrow a^2 = 0^2 \Rightarrow a^2 = 0$$

$$(d) x^3 = x$$

$$\Leftrightarrow x^3 - x = 0 \stackrel{F}{\Leftrightarrow} x(x^2 - 1) = 0 \stackrel{F}{\Leftrightarrow} x(x-1)(x+1) = 0$$
$$\Leftrightarrow \begin{cases} x=0 \\ x-1=0 \\ x+1=0 \end{cases} \Leftrightarrow \begin{cases} x=0 \\ x=1 \\ x=-1 \end{cases}$$
$$S : \{-1, 0, 1\}$$

$$(e) (12x^4 - 3x^2) + (12x^3 - 3x) = 0$$

$$\stackrel{F}{\Leftrightarrow} 3x^2(4x^2 - 1) + 3x(4x^2 - 1) = 0$$
$$\Leftrightarrow (4x^2 - 1)(3x^2 + 3x) = 0$$
$$\stackrel{F}{\Leftrightarrow} (2x-1)(2x+1)(3x)(x+1) = 0$$
$$\Leftrightarrow \begin{cases} 3x=0 \\ 2x-1=0 \\ 2x+1=0 \\ x+1=0 \end{cases} \Leftrightarrow \begin{cases} x=0 \\ x=\frac{1}{2} \\ x=-\frac{1}{2} \\ x=-1 \end{cases}$$

$$S : \left\{ -1, -\frac{1}{2}, 0, \frac{1}{2} \right\}$$

$$(f) 2x(x^2 - 1) = 3(x^2 - 1)$$

$$\Leftrightarrow 2x(x^2 - 1) - 3(x^2 - 1) = 0$$

$$\text{F} \Leftrightarrow (x^2 - 1)(2x - 3) = 0$$

$$\text{F} \Leftrightarrow (x - 1)(x + 1)(2x - 3) = 0$$

$$\Leftrightarrow \begin{cases} x - 1 = 0 \\ x + 1 = 0 \\ 2x - 3 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 1 \\ x = -1 \\ x = \frac{3}{2} \end{cases}$$

$$S: \left\{-1, 1, \frac{3}{2}\right\}$$

$$(g) x^2(4x - 1) + 9(1 - 4x) = 0 \Leftrightarrow x^2(4x - 1) - 9(4x - 1) = 0$$

$$\text{F} \Leftrightarrow (4x - 1)(x^2 - 9) = 0$$

$$\text{F} \Leftrightarrow (4x - 1)(x - 3)(x + 3) = 0$$

$$\Leftrightarrow \begin{cases} 4x - 1 = 0 \\ x - 3 = 0 \\ x + 3 = 0 \end{cases} \Leftrightarrow \begin{cases} x = \frac{1}{4} \\ x = 3 \\ x = -3 \end{cases}$$

$$S: \left\{-3, \frac{1}{4}, 3\right\}$$

$$(h) (3x - 1)(x + 2) = x(x + 2)$$

$$\Leftrightarrow (3x - 1)(x + 2) - x(x + 2) = 0$$

$$\text{F} \Leftrightarrow (x + 2)[(3x - 1) - x] = 0$$

$$\Leftrightarrow (x + 2)(2x - 1) = 0$$

$$\Leftrightarrow \begin{cases} x + 2 = 0 \\ 2x - 1 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -2 \\ x = \frac{1}{2} \end{cases}$$

$$S: \left\{-2, \frac{1}{2}\right\}$$

$$(i) (x-1)(3x-2) = 4x(2-3x)$$

$$\Leftrightarrow (x-1)(3x-2) - 4x(2-3x) = 0$$

$$\Leftrightarrow (x-1)(3x-2) + 4x(3x-2) = 0$$

$$\Leftrightarrow (3x-2)[(x-1)+4x] = 0$$

$$\Leftrightarrow (3x-2)(5x-1) = 0$$

$$\Leftrightarrow \begin{cases} 3x-2=0 \\ 5x-1=0 \end{cases} \Leftrightarrow \begin{cases} x=\frac{2}{3} \\ x=\frac{1}{5} \end{cases} \quad S: \left\{ \frac{1}{5}, \frac{2}{3} \right\}$$

$$(j) 2x^3 + x^2 + 2x = -1$$

$$\Leftrightarrow (2x^3 + x^2) + (2x + 1) = 0$$

$$\Leftrightarrow x^2(2x+1) + (2x+1) = 0$$

$$\Leftrightarrow (2x+1)(x^2+1) = 0$$

$$\Leftrightarrow \begin{cases} 2x+1=0 \\ x^2+1=0 \end{cases} \Leftrightarrow \begin{cases} x=-\frac{1}{2} \\ x^2+1=0 \end{cases} \rightarrow \text{imp}$$

$$S: \left\{ -\frac{1}{2} \right\}$$

3. Résoudre les équations fractionnaires suivantes :

$$(a) \frac{2}{x-3} - \frac{1}{x} = \frac{1}{x^2-3x} \Leftrightarrow \frac{2}{x-3} - \frac{1}{x} = \frac{1}{x(x-3)}$$

CE: $x \neq 0, x \neq 3$

$$\Leftrightarrow \frac{2x - (x-3)}{x(x-3)} = \frac{1}{x(x-3)}$$

$$\Leftrightarrow x + 3 = 1 \Leftrightarrow x = -2$$

$$S: \{-2\}$$

$$(b) \frac{3}{x-1} - \frac{1}{x+2} = \frac{-5}{1-x} \quad \text{CE : } x \neq 1, x \neq -2$$

$$\Leftrightarrow \frac{3(x+2) - (x-1)}{(x-1)(x+2)} = \frac{5(x+2)}{(x-1)(x+2)} \quad (*)$$

$$\Leftrightarrow 3x + 6 - x + 1 = 5x + 10$$

$$\Leftrightarrow -3x = 3$$

$$\Leftrightarrow x = -1$$

$$S: \{-1\}$$

$$(*) \text{ car } \frac{-5}{1-x} = \frac{5}{x-1}$$

$$(c) 1 - \frac{x}{x-2} + \frac{1}{x+2} = 0$$

$\subseteq : n \neq 2, n \neq -2$

$$\Leftrightarrow \frac{(n-2)(n+2) - x(n+2) + (n-2)}{(n-2)(n+2)} = 0$$

$$\Leftrightarrow x^2 - 4 - xn^2 - 2n + n - 2 = 0$$

$$\Leftrightarrow -n - 6 = 0$$

$$\Leftrightarrow n = -6$$

$$S: \{-6\}$$

$$(d) \frac{-1}{x+3} - \frac{2}{x-1} = \frac{1}{(x+3)(x-1)}$$

$\subseteq : n \neq 1, n \neq -3$

$$\Leftrightarrow \frac{-(n-1) - 2(n+3)}{(n+3)(n-1)} = \frac{1}{(n+3)(n-1)}$$

$$\Leftrightarrow -n + 1 - 2n - 6 = 1$$

$$\Leftrightarrow -3n = 6$$

$$\Leftrightarrow n = -2$$

$$S: \{-2\}$$

$$(e) \frac{1}{x-1} + \frac{2}{x+1} = \frac{1}{x^2 - 2x + 1} \Leftrightarrow \frac{1}{(x-1)} + \frac{2}{(x+1)} = \frac{1}{(x-1)^2}$$

C6: $x \neq 1, m \neq -1$

$$\Leftrightarrow \frac{(m-1)(m+1) + 2(m-1)^2}{(x-1)^2(x+1)} = \frac{(m+1)}{(x-1)^2(x+1)}$$

$$\Leftrightarrow x^2 - 1 + 2(x^2 - 2x + 1) = m+1$$

$$\Leftrightarrow 3x^2 - 5x = 0$$

$$\Leftrightarrow x(3x-5) = 0$$

$$\Leftrightarrow \begin{cases} m=0 \\ 3x-5=0 \end{cases} \Leftrightarrow \begin{cases} x=0 \\ m=\frac{5}{3} \end{cases}$$

$$S: \left\{ 0, \frac{5}{3} \right\}$$

$$(f) \frac{2}{(3x+2)(7x-1)} = \frac{1}{3x+2} + \frac{3}{7x-1} \quad \text{C6 } m \neq -\frac{2}{3}, n \neq \frac{1}{2}$$

$$\Leftrightarrow \frac{2}{(3x+2)(7x-1)} = \frac{7x-1 + 3(3x+2)}{(3x+2)(7x-1)}$$

$$\Leftrightarrow 2 = 7x-1 + 9x+6$$

$$\Leftrightarrow -3 = 16x$$

$$\Leftrightarrow x = -\frac{3}{16}$$

$$S: \left\{ -\frac{3}{16} \right\}$$

$$(g) \frac{2}{x-3} - \frac{1}{x+1} = \frac{4}{(x-3)(x+1)} - \frac{2}{x-3} \quad \underline{\text{CE}} : x \neq -1, x \neq 3$$

$$\Leftrightarrow \frac{2(x+1) - (x-3)}{(x+1)(x-3)} = \frac{4 - 2(x+1)}{(x+1)(x-3)}$$

$$\Leftrightarrow 2x + 2 - x + 3 = 4 - 2x - 2$$

$$\Leftrightarrow 3x = -3$$

$$\Leftrightarrow x = -1 \quad (\text{A.R. von CE})$$

$S: \emptyset$

$$(h) \frac{3}{x-2} - \frac{1}{2x+5} + \frac{2}{2x^2+x-10} = 0$$

Hinweis

$$\Leftrightarrow \frac{3}{x-2} - \frac{1}{2x+5} + \frac{2}{(2x+5)(x-2)} = 0$$

$$\underline{\text{CE}} : x \neq 2, x \neq -\frac{5}{2}$$

$$\Leftrightarrow \frac{3(2x+5) - (x-2) + 2}{(x-2)(2x+5)} = 0$$

$$\Leftrightarrow 6x + 15 - x + 2 + 2 = 0$$

$$\Leftrightarrow 5x + 19 = 0$$

$$\Leftrightarrow x = -\frac{19}{5}$$

$S: \left\{ -\frac{19}{5} \right\}$

$$(i) \frac{1}{4x-3} - \frac{2}{2x+1} = \frac{3}{(4x-3)(2x+1)} \quad \text{C.E. } n \neq \frac{3}{4}, n \neq -\frac{1}{2}$$

$$\Leftrightarrow \frac{(2x+1) - 2(4n-3)}{(4n-3)(2x+1)} = \frac{3}{(4n-3)(2x+1)}$$

$$\Leftrightarrow 2n + 1 - 8n + 6 = 3$$

$$\Leftrightarrow -6n = -4$$

$$\Leftrightarrow n = \frac{2}{3}$$

$$S : \left\{ \frac{2}{3} \right\}$$

$$(j) \frac{2}{3x-1} - \frac{-1}{2x-1} - \frac{1}{(2x-1)(3x-1)} = 0 \quad \text{C.E. } x \neq \frac{1}{3}, x \neq \frac{1}{2}$$

$$\Leftrightarrow \frac{2(2x-1) - (-1)(3x-1) - 1}{(3x-1)(2x-1)} = 0$$

$$\Leftrightarrow 4x - 2 + 3x - 1 - 1 = 0$$

$$\Leftrightarrow 7x = 4$$

$$\Leftrightarrow x = \frac{4}{7}$$

$$S : \left\{ \frac{4}{7} \right\}$$

4. Résoudre les équations et inéquations suivantes :

$$(a) \frac{7x^2 - 3x + 1}{3x - 8} = 2x - 3$$

$$\text{CE : } n \neq \frac{8}{3}$$

$$\Leftrightarrow \frac{7x^2 - 3n + 1}{3n - 8} = \frac{(2n - 3)(3n - 8)}{3n - 8}$$

$$\Leftrightarrow 7x^2 - 3n + 1 = 6x^2 - 16n - 9n + 24$$

$$\Leftrightarrow x^2 + 22n - 23 = 0$$

Mais

$$\Leftrightarrow (n - 1)(n + 23) = 0$$

$$\Leftrightarrow \begin{cases} n - 1 = 0 \\ n + 23 = 0 \end{cases} \Leftrightarrow \begin{cases} n = 1 \\ n = -23 \end{cases}$$

$$S : \{-23, 1\}$$

$$(b) \frac{x^4 - 7x^3 - 8x^2}{5x^3 + 2x - 7} = 0$$

$$\begin{aligned} & \text{Case } 5n^3 + 2n - 7 \neq 0 \\ & \text{Hence } (n-1)(5n^2 + 5n + 7) \neq 0 \\ & \Leftrightarrow n \neq 1 \end{aligned}$$

$$\Leftrightarrow x^4 - 7x^3 - 8x^2 = 0$$

$$\Leftrightarrow x^2(x^2 - 7x - 8) = 0$$

$$\begin{aligned} & \text{Hence} \\ & \Leftrightarrow x^2(x+1)(x-8) = 0 \end{aligned}$$

$$\Leftrightarrow \begin{cases} x^2 = 0 \\ x+1 = 0 \\ x-8 = 0 \end{cases} \quad \Rightarrow \begin{cases} x = 0 \\ x = -1 \\ x = 8 \end{cases}$$

$$S: \{-1, 0, 8\}$$

$$(c) \frac{(x-1)^3}{x^2} = x-3$$

CE : $x \neq 0$

$$\Leftrightarrow \frac{(x-1)^3}{x^2} = \frac{x^2(x-3)}{x^2}$$

$$\Leftrightarrow x^3 - 3x^2 + 3x - 1 = x^3 - 3x^2$$

$$\Leftrightarrow 3x - 1 = 0$$

$$\Leftrightarrow x = \frac{1}{3}$$

$$S. \left\{ \frac{1}{3} \right\}$$

$$(d) x+1 = \frac{1-2x}{x+1} \quad \underline{\text{CE}} \quad n \neq -1$$

$$\Leftrightarrow \frac{(n+1)^2}{n+1} = \frac{1-2n}{n+1}$$

$$\Leftrightarrow n^2 + 2n + 1 = 1 - 2n$$

$$\Leftrightarrow n^2 + 4n = 0$$

$$\Leftrightarrow n(n+4) = 0$$

$$\Leftrightarrow \begin{cases} n = 0 \\ n+4 = 0 \end{cases} \quad \Leftrightarrow \begin{cases} n = 0 \\ n = -4 \end{cases}$$

$$S: \{-4, 0\}$$

$$(e) \frac{2x-1}{x-2} \cdot \frac{4x-1}{x+2} = 1 \quad \underline{\text{CE}} \quad n \neq 1, n \neq -2$$

$$\Leftrightarrow \frac{(2n-1)(n+2) - (4n-1)(n-2)}{(n-2)(n+2)} = \frac{(n+2)(n-2)}{(n+2)(n-2)}$$

$$\Leftrightarrow 2n^2 + 4n - n - 2 - (4n^2 - 8n - n + 2) = n^2 - 4$$

$$\Leftrightarrow n^2 + 3n + 2 - 4n^2 + 9n - 2 = 0$$

$$\Leftrightarrow -3n^2 + 12n = 0$$

$$\Leftrightarrow -3n(n-4) = 0$$

$$\Leftrightarrow \begin{cases} -3n = 0 \\ n-4 = 0 \end{cases} \quad \Leftrightarrow \begin{cases} n = 0 \\ n = 4 \end{cases}$$

$$S: \{0, 4\}$$

$$(f) \frac{-12}{x-4} \geq x+3$$

(c6: $x-4 \neq 0 \Leftrightarrow x \neq 4$)

$$\Leftrightarrow -\frac{12}{x-4} - (x+3) \geq 0$$

$$\Leftrightarrow \frac{-12 - (x-4)(x+3)}{x-4} \geq 0$$

$$\Leftrightarrow \frac{-12 - (x^2 - 4x + 3x - 12)}{x-4} \geq 0$$

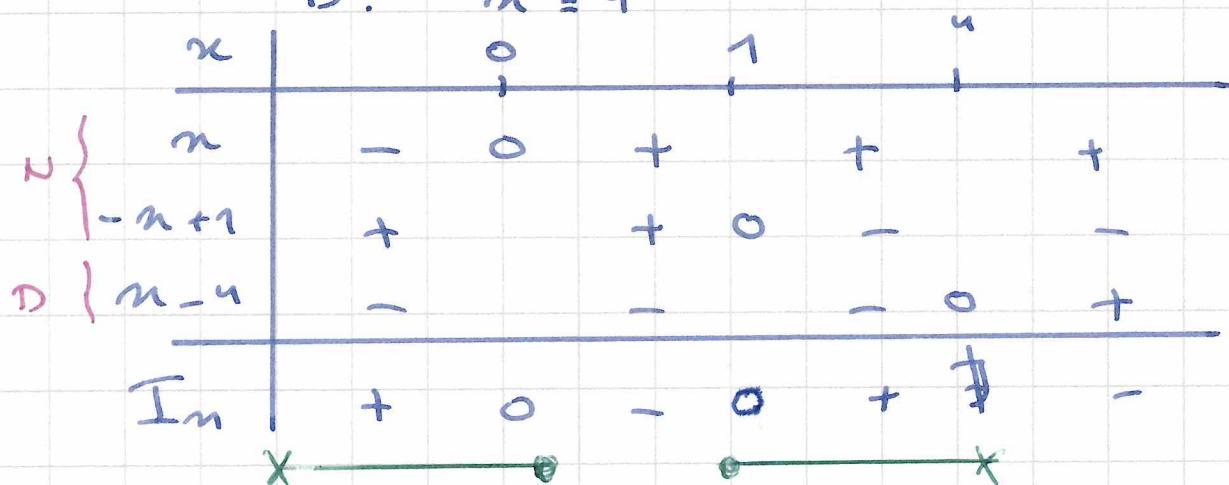
$$\Leftrightarrow \frac{-12 - x^2 + x + 12}{x-4} \geq 0$$

$$\Leftrightarrow \frac{-x^2 + x}{x-4} \geq 0$$

$$\Leftrightarrow \frac{x(-x+1)}{x-4} \geq 0$$

Zeros $\frac{N}{D}$: $x=0 ; x=1$

$D: x=4$



$$S: (-\infty, 0] \cup [1, 4]$$

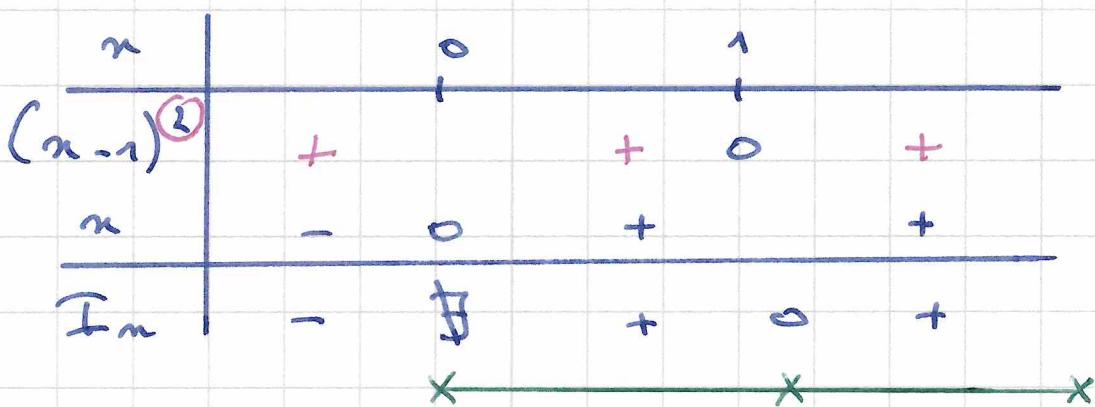
$$(g) x + \frac{1}{x} > 2$$

$$\Leftrightarrow x + \frac{1}{x} - 2 > 0 \Leftrightarrow \frac{x^2 + 1 - 2x}{x} > 0$$

$$\Leftrightarrow \frac{(x-1)^2}{x} > 0$$

zeros: N: $x=1$

D: $x=0$



$$S:]0, 1[\cup]1, +\infty$$

$$\text{ou } S: \mathbb{R}_0^+ \setminus \{1\}$$

$$(h) \frac{2x-3}{x-1} \leq x-1$$

$$\Leftrightarrow \frac{2x-3}{x-1} - x-1 \leq 0$$

$$\Leftrightarrow \frac{2x-3 - (x-1)^2}{x-1} \leq 0$$

$$\Leftrightarrow \frac{2x-3 - (x^2-2x+1)}{(x-1)} \leq 0$$

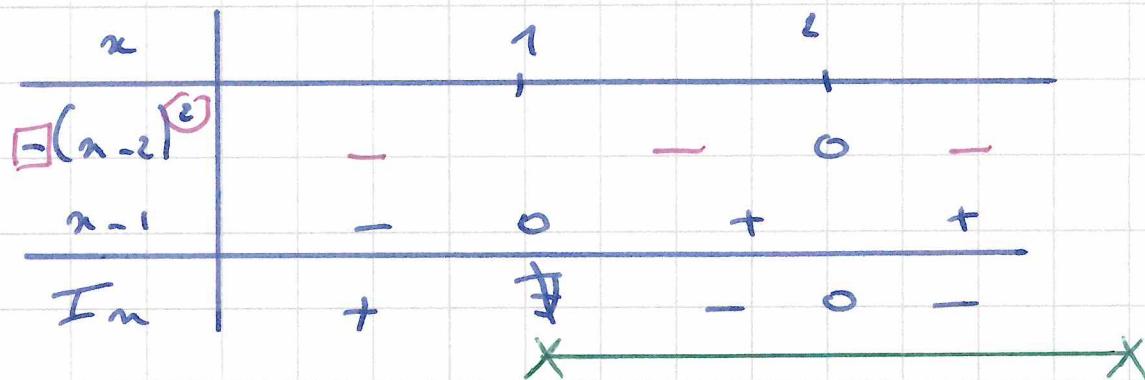
$$\Leftrightarrow \frac{-x^2+4x-4}{x-1} \leq 0$$

$$\Leftrightarrow \frac{-(x^2-4x+4)}{x-1} \leq 0$$

$$\Leftrightarrow \frac{-(x-2)^2}{x-1} \leq 0$$

Zeichn: N: $x=2$

D: $x=1$



S: $]1, +\infty$

$$(i) 1 + \frac{1}{x+2} \leq \frac{x}{x-2}$$

$$\Leftrightarrow 1 + \frac{1}{x+2} - \frac{x}{x-2} \leq 0$$

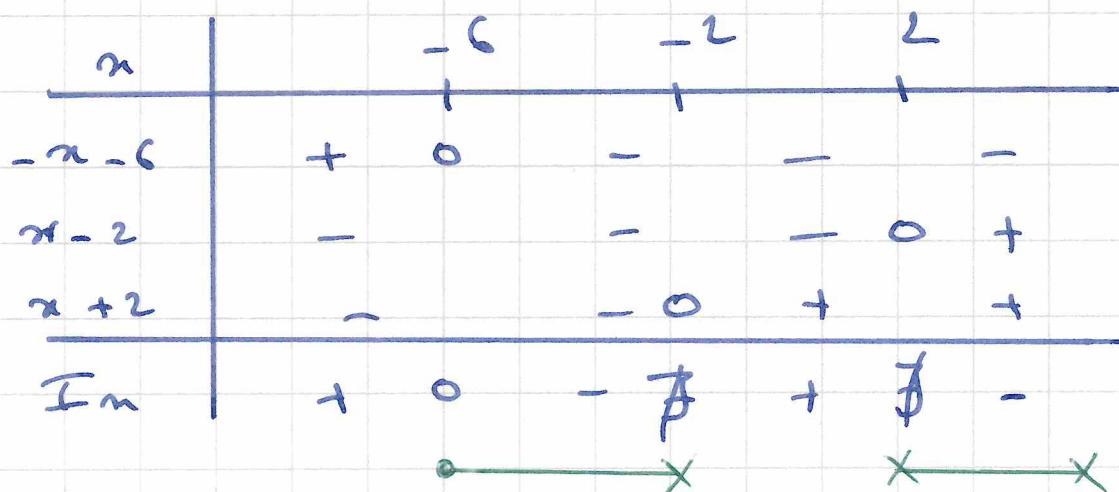
$$\Leftrightarrow \frac{(x+2)(x-2) + (x-2) - x(x+2)}{(x-2)(x+2)} \leq 0$$

$$\Leftrightarrow \frac{x^2 - 4 + x - 2 - x^2 - 2x}{D} \leq 0$$

$$\Leftrightarrow \frac{-x-6}{(x-2)(x+2)} \leq 0$$

Zeilen: N : $x = -6$

D : $x = -2, x = 2$



$$S: [-6, -2] \cup [2, +\infty)$$

$$(j) \frac{2}{(3x+2)(7x-1)} > \frac{1}{3x+2} + \frac{3}{7x-1}$$

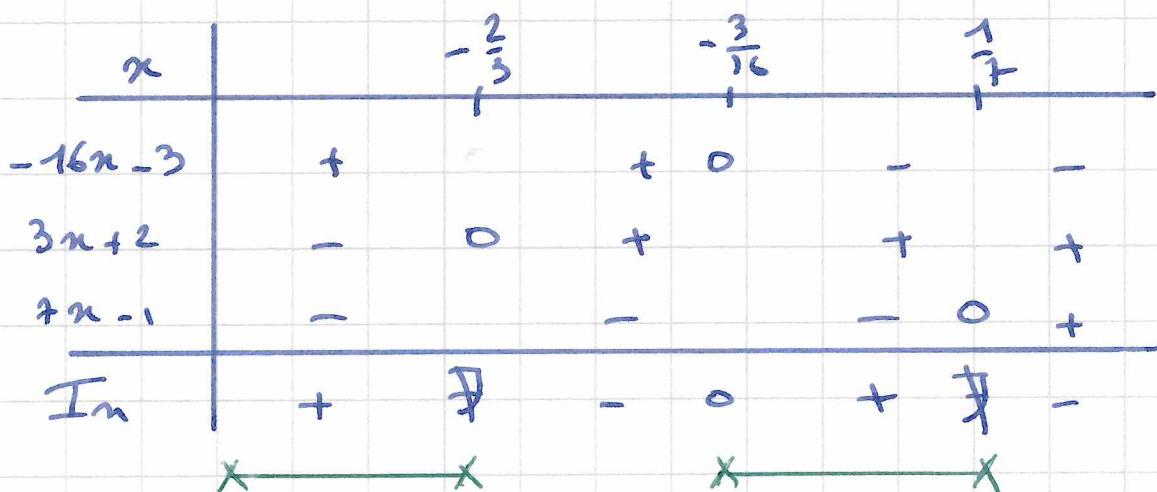
$$\Leftrightarrow \frac{2}{(3x+2)(7x-1)} - \frac{1}{3x+2} - \frac{3}{7x-1} > 0$$

$$\Leftrightarrow \frac{2 - (7x-1) - 3(3x+2)}{(3x+2)(7x-1)} > 0$$

$$\Leftrightarrow \frac{-16x - 3}{(3x+2)(7x-1)} > 0$$

Zeilen: N: $x = -\frac{3}{16}$

D: $x = -\frac{2}{3}, x = \frac{1}{7}$



S: $-\infty, -\frac{2}{3} [\cup] -\frac{3}{16}, \frac{1}{7} [$

$$(k) \frac{3}{x-2} + \frac{2}{2x^2+x-10} \geq \frac{1}{2x+5}$$

\hookrightarrow Mittern: $2x^2+x-10 = (x-2)(2x+5)$

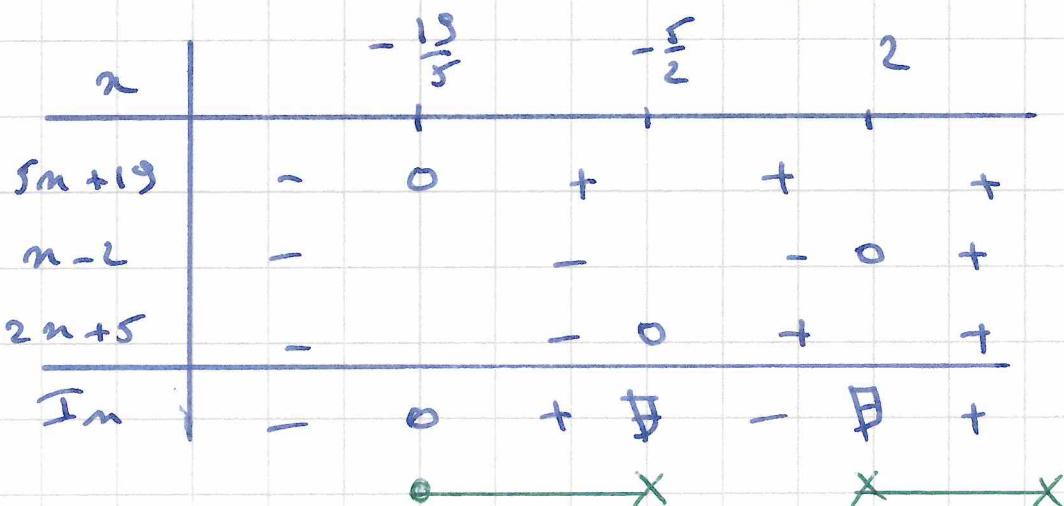
$$\Leftrightarrow \frac{3}{x-2} + \frac{2}{(x-2)(2x+5)} - \frac{1}{2x+5} \geq 0$$

$$\Leftrightarrow \frac{3(2x+5) + 2 - (x-2)}{(x-2)(2x+5)} \geq 0$$

$$\Leftrightarrow \frac{5x+19}{(x-2)(2x+5)} \geq 0$$

Zeilen: N: $x = -\frac{19}{5}$

D: $x = 2, x = -\frac{5}{2}$



S: $[-\frac{19}{5}, -\frac{5}{2}] \cup [2, +\infty)$

$$(1) \frac{x-1}{x} + \frac{x}{x-2} < \frac{4}{x^2 - 2x}$$

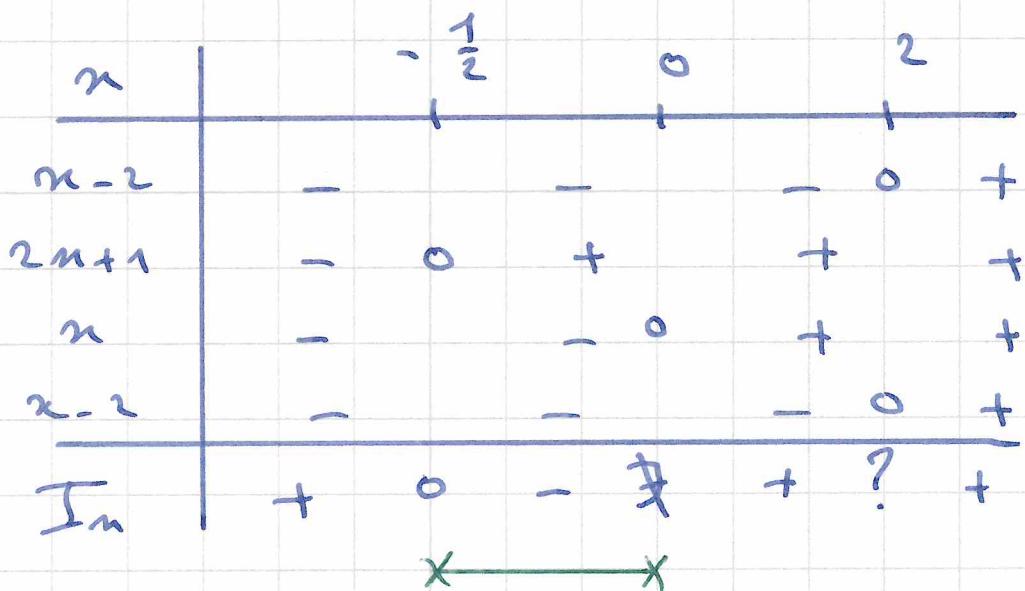
$$\Leftrightarrow \frac{x-1}{x} + \frac{x}{x-2} - \frac{4}{x(x-2)} < 0$$

$$\Leftrightarrow \frac{(x-1)(x-2) + x^2 - 4}{x(x-2)} < 0$$

$$\Leftrightarrow \frac{x^2 - 3x + 2 + x^2 - 4}{x(x-2)} < 0$$

$$\Leftrightarrow \frac{2x^2 - 3x - 2}{x(x-2)} < 0$$

zeros N: $x = 2, x = -\frac{1}{2}$ (Horner)
D: $x=0, x=2$ $(x-2)(2x+1)$



$$S:]-\frac{1}{2}, 0[$$

$$(m) \frac{x+1}{2x+4} + \frac{1}{x+1} < \frac{1}{x^2+3x+2}$$

↳ Horner (n+2)(n+1)

$$\Leftrightarrow \frac{x+1}{2(n+2)} + \frac{1}{n+1} - \frac{1}{(n+2)(n+1)} < 0$$

$$\Leftrightarrow \frac{(n+1)(n+1) + 2(n+2) - 2}{2(n+2)(n+1)} < 0$$

$$\Leftrightarrow \frac{n^2 + 4n + 1 + 2n + 4 - 2}{D} < 0$$

$$\Leftrightarrow \frac{n^2 + 4n + 3}{2(n+2)(n+1)} < 0$$

Zeile: N: $x = -3, n = -1$ (Horner)
 $(n+3)(n+1)$

$$D: n = -2, x = -1$$

x	-3	-2	-1
$n+3$	-	+	+
$n+1$	-	-	0
$n+2$	-	-	+
$x+1$	-	-	+
I_n	+	0	-

\times

$$S:]-3, -2[$$

$$(n) \frac{2}{3x-1} - \frac{-1}{2x-1} > \frac{1}{(3x-1)(2x-1)}$$

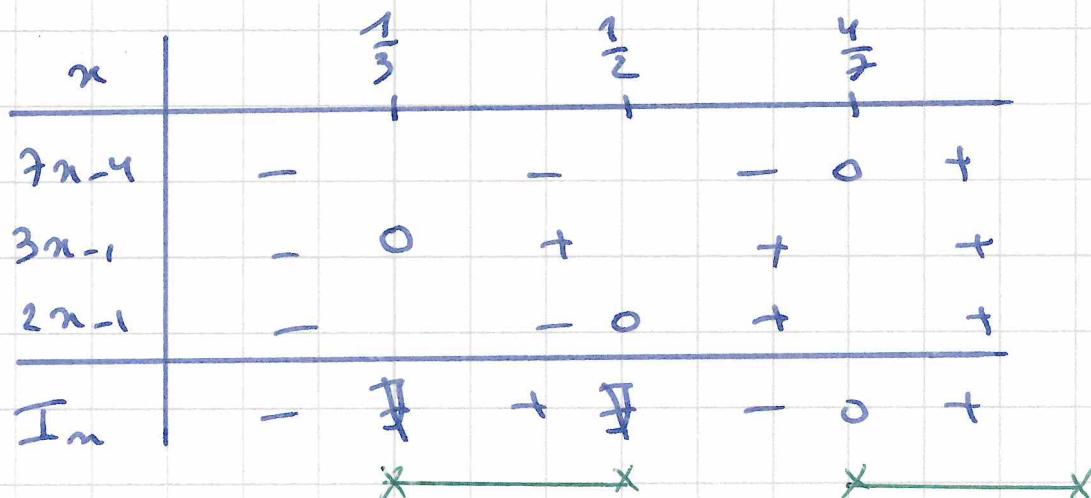
$$\Leftrightarrow \frac{2}{3x-1} + \frac{+1}{2x-1} - \frac{1}{(3x-1)(2x-1)} > 0$$

$$\Leftrightarrow \frac{2(2x-1) + (3x-1) - 1}{(3x-1)(2x-1)} > 0$$

$$\Leftrightarrow \frac{7x-4}{(3x-1)(2x-1)} > 0$$

zeile: N: $x = \frac{4}{7}$

$$D: x = \frac{1}{2}, x = \frac{1}{3}$$



S: $\left] \frac{1}{3}, \frac{1}{2} \right[\cup \left] \frac{4}{7}, +\infty \right)$

$$(o) \frac{-1}{x+3} - \frac{2}{x-1} \geq \frac{1}{x^2 + 2x - 3}$$

\hookrightarrow Manner: $(x+3)(x-1)$

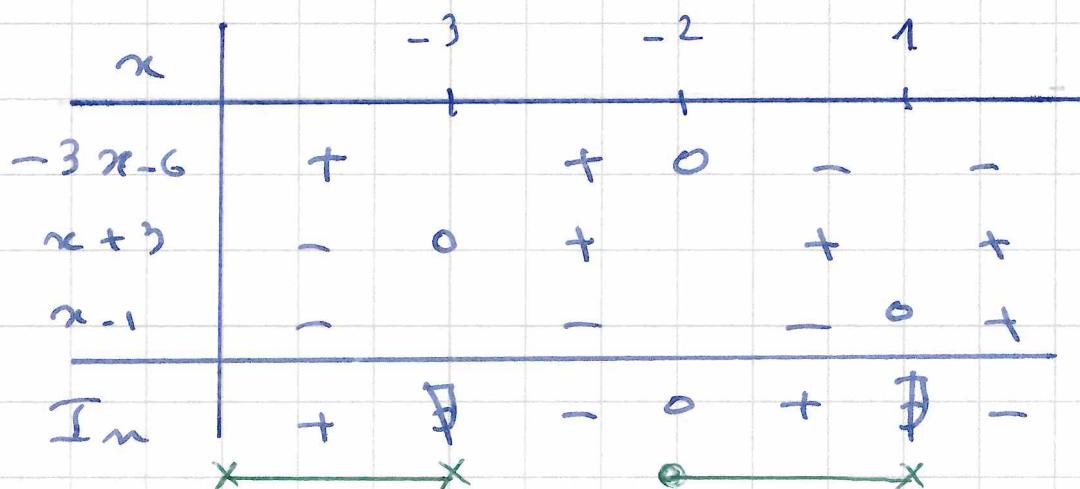
$$\Leftrightarrow \frac{-1}{x+3} - \frac{2}{x-1} - \frac{1}{(x+3)(x-1)} \geq 0$$

$$\Leftrightarrow \frac{-(x-1) - 2(x+3) - 1}{(x+3)(x-1)} \geq 0$$

$$\Leftrightarrow \frac{-3x - 6}{(x+3)(x-1)} \geq 0$$

gerade $N \quad n = -2$

D $n = 1, \quad n = -3$



$$S: -\infty, -3 \cup [-2, 1]$$