

Semaine 3 : Factoriser en utilisant les produits remarquables (une mise en évidence préalable peut parfois être nécessaire) :

$$1. 2a - 8a^3 + 8a^5 = 2a(1 - 4a^2 + 4a^4) \\ = 2a(1 - 2a^2)^2$$

$$2. 64x^5 + 16x^4 + x^3 = x^3(64x^2 + 16x + 1) \\ = x^3(8x + 1)^2$$

$$3. x^2 - 2 + \frac{1}{x^2} \quad a^3 - 4a^9 = a^3(1 - 4a^6) \\ = a^3(1 - 2a^3)(1 + 2a^3)$$

$$4. 100a^2 + 9b^6 - 30ab^3 \quad 25x^5y - 49xy^3 = xy(25x^4 - 49y^2) \\ = xy(5x^2 - 7y)(5x^2 + 7y)$$

$$5. 25x^4 + 16x^2y^4 - 40x^3y^2 = x^2(25x^2 + 16y^4 - 40xy^2) \\ = x^2(5x - 4y^2)^2$$

$$6. 4a^5 + 12a^3b^4 + 9ab^8 = a(4a^4 + 12a^2b^4 + 9b^8) \\ = a(2a^2 + 3b^4)^2$$

$$7. (2a + 1)^2 - (3 - a)^2 = [(2a + 1) - (3 - a)][(2a + 1) + (3 - a)] \\ = (3a - 2)(a + 4)$$

$$8. 9 + \frac{a^2b^2}{4} - 3ab = \left(3 - \frac{ab}{2}\right)^2$$

$$9. 25a^2 - (b - 2a)^2 = x^2 + 12x - 36 = -(x^2 - 12x + 36) \\ = -(x - 6)^2$$

Factoriser en utilisant la méthode des diviseurs binômes :

1. $x^2 + 10x + 16$

div 16 : $\{\pm 1, \pm 2, \pm 4, \pm 8, \pm 16\}$

$P(1) \neq 0, P(-1) \neq 0, P(2) \neq 0, P(-2) = 0$

	1	10	16
-2		-2	-16
	1	8	0

$$x^2 + 10x + 16 = (x + 2)(x + 8)$$

2. $x^2 - 4x - 32$

div 32 : $\{\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32\}$

$P(-4) = 0$

	1	-4	-32
-4		-4	32
	1	-8	0

$$x^2 - 4x - 32 = (x + 4)(x - 8)$$

3. $x^2 + 5x - 14$

div 14: $\{ \pm 1, \pm 2, \pm 7, \pm 14 \}$

$P(x) = 0$

	1	5	-14
(2)		2	14
	1	7	0

$x^2 + 5x - 14 = (x - 2)(x + 7)$

4. $x^2 + 20x + 19$

div 19: $\{ \pm 1, \pm 19 \}$

$P(x) = 0$

	1	20	19
(-1)		-1	-19
	1	19	0

$x^2 + 20x + 19 = (x + 1)(x + 19)$

$$5. x^2 - 4x - 12$$

$$\text{div } 12: \{ \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12 \}$$

$$P(-2) = 0$$

	1	-4	-12
-2		-2	12
	1	-6	0

$$x^2 - 4x - 12 = (x + 2)(x - 6)$$

$$6. x^4 + 5x^3 - 15x^2 - 45x + 54$$

$$\text{div } 54: \{ \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm 27, \pm 54 \}$$

$$P(1) = 0, P(3) = 0, P(-3) = 0, P(-6) = 0$$

	1	5	-15	-45	54
1		1	6	-9	-54
	1	6	-9	-54	0
3		3	27	54	
	1	9	18	0	
-3		-3	-18		
	1	6	0		

$$x^4 + 5x^3 - 15x^2 - 45x + 54 = (x - 1)(x - 3)(x + 3)(x + 6)$$

$$7. x^4 + 2x^3 - 16x^2 - 2x + 15$$

$$\text{div } 15: \{ \pm 1, \pm 3, \pm 5, \pm 15 \}$$

$$P(1)=0, P(-1)=0, P(3)=0, P(-5)=0$$

	1	2	-16	-2	15
1		1	3	-13	-15
	1	3	-13	-15	0
-1		-1	-2	15	
	1	2	-15		
3		3	15		
	1	5	0		

$$x^4 + 2x^3 - 16x^2 - 2x + 15 = (x-1)(x+1)(x-3)(x+5)$$

$$8. x^4 - 7x^3 + 17x^2 - 17x + 6$$

$$\text{div } 6: \{ \pm 1, \pm 2, \pm 3, \pm 6 \}$$

$$P(1)=0, P(2)=0, P(3)=0$$

	1	-7	17	-17	6
1		1	-6	11	-6
	1	-6	11	-6	0
1		1	-5	6	
	1	-5	6	0	
2		2	-6		
	1	-3	0		

$$x^4 - 7x^3 + 17x^2 - 17x + 6 = (x-1)^2(x-2)(x-3)$$

9. $x^5 + 3x^4 - 16x - 48$

$P(48) = \{ \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16, \pm 24, \pm 48 \}$

$P(2) = 0, P(-2) = 0, P(-3) = 0$

	1	3	0	0	-16	-48
(2)		2	10	20	40	48
	1	5	10	20	24	0
(-2)		-2	-6	-8	-24	
	1	3	4	12	0	
(-3)		-3	0	-12		
	1	0	4	0		

$x^5 + 3x^4 - 16x - 48 = (x - 2)(x + 2)(x + 3)(x^2 + 4)$