

Semaine 4 : Produits remarquables du 3^{ème} degré :

Développer

$$\begin{aligned} 1. (3a + b)^3 &= (3a)^3 + 3 \cdot (3a)^2 \cdot b + 3 \cdot 3a \cdot b^2 + b^3 \\ &= 27a^3 + 27a^2b + 9ab^2 + b^3 \end{aligned}$$

$$\begin{aligned} 2. (a - 5)^3 &= a^3 + 3a^2(-5) + 3a(-5)^2 + (-5)^3 \\ &= a^3 - 15a^2 + 75a - 125 \end{aligned}$$

$$\begin{aligned} 3. (2a + 3b)^3 &= (2a)^3 + 3(2a)^2(3b) + 3(2a)(3b)^2 + (3b)^3 \\ &= 8a^3 + 36a^2b + 54ab^2 + 27b^3 \end{aligned}$$

$$\begin{aligned} 4. (4 - 3a)^3 &= 4^3 - 3(4)^2(3a) + 3(4)(3a)^2 - (3a)^3 \\ &= 64 - 144a + 108a^2 - 27a^3 \end{aligned}$$

$$\begin{aligned} 5. \left(x + \frac{1}{x}\right)^3 &= x^3 + 3x^2 \frac{1}{x} + 3x \frac{1}{x^2} + \frac{1}{x^3} \\ &= x^3 + 3x + \frac{3}{x} + \frac{1}{x^3} \end{aligned}$$

$$6. \left(\frac{x}{2} + \frac{x^2}{3}\right)^3 = \left(\frac{x}{2}\right)^3 + 3\left(\frac{x}{2}\right)^2\left(\frac{x^2}{3}\right) + 3\left(\frac{x}{2}\right)\left(\frac{x^2}{3}\right)^2 + \left(\frac{x^2}{3}\right)^3$$

$$= \frac{x^3}{8} + \frac{x^4}{2} + \frac{x^5}{6} + \frac{x^6}{27}$$

$$7. \left(2x - \frac{3}{2}x\right)^3 = (2x)^3 - 3(2x)^2\left(\frac{3}{2}x\right) + 3(2x)\left(\frac{3}{2}x\right)^2 - \left(\frac{3}{2}x\right)^3$$

$$= 8x^3 - 18x^3 + 27x^3 - \frac{27}{8}x^3$$

$$= \frac{x^3}{8}$$

Factoriser

$$1. x^3 - 8y^3 = (x - 2y)(x^2 + 2xy + 4y^2)$$

$$2. 64a^3 + 125b^3 = (8a + 5b)(64a^2 - 40ab + 25b^2)$$

$$3. x^3 - \frac{27}{x^6} = \left(x - \frac{3}{x^2}\right)\left(x^2 + \frac{3}{x} + \frac{9}{x^4}\right)$$

$$4. \frac{a^3}{125} - 3\frac{a^2b}{50} + 3\frac{ab^2}{20} - \frac{b^3}{8} = \left(\frac{a}{5} - \frac{b}{2}\right)^3$$

$$5. \frac{8x^3}{27} + \frac{x^2y}{3} + \frac{xy^2}{8} + \frac{y^3}{64} = \left(\frac{2x}{3} + \frac{y}{4}\right)^3$$

$$6. 216a^6 + b^3 = (6a^2 + b)(36a^4 + 6a^2b + b^2)$$

$$7. \frac{64}{x^{12}} - 729x^6 = \left(\frac{4}{x^4} - 9x^2\right)\left(\frac{16}{x^8} + \frac{36}{x^2} + 81x^4\right)$$

$$= \left(\frac{2}{x^2} - 3x\right)\left(\frac{2}{x^2} + 3x\right)\left(\frac{16}{x^8} + \frac{36}{x^2} + 81x^4\right)$$

$$8. b^3 + 6b^2 + 12b + 8 = (b + 2)^3$$

$$9. 8a^3 - 60a + \frac{150}{a} - \frac{125}{a^3} = \left(2a - \frac{5}{a}\right)^3$$