

Drill factorisation : Solutions

Semaine 1 : Factoriser en utilisant la mise en évidence :

$$1. ab - b^2 = b(a - b)$$

$$2. xy + y = y(x + 1)$$

$$3. a^2b - ab^2 = ab(a - b)$$

$$4. xy - 2y = y(x - 2)$$

$$5. x^3y^2 - x^2y^3 = x^2y^2(x - y)$$

$$6. 6x^2y + 4xy^2 = 2xy(3x + 2y)$$

$$7. 15x^7b^2 - 10x^5b^3 = 5x^5b^2(3x^2 - 2b)$$

$$8. y(b - a) + b(b - a) = (b - a)(y + b)$$

$$9. 12a^2b^2 - 18ab^3 + 24a^3b = 6ab(2ab - 3b^2 + 4a^2)$$

$$10. 12x^2y^3 - 30x^3y^2 + 18xy^4 = 6xy^2(2xy - 5x^2 + 3y^2)$$

$$11. 3x^2 - 9xy^2 + 6x = 3x(x - 3y^2 + 2)$$

$$12. 3xyz^3 - 21x^2y^2z^2 - 6x^3y^3z = 3xyz(z^2 - 7xy - 2x^2y^2)$$

$$13. 5a(a+2)^2 - 3a^2(a+2) = a(a+2)[5(a+2) - 3a]$$

$$= a(a+2)(2a+10) = 2a(a+2)(a+5)$$

$$14. 5(x+y-z) - 10x(x+y-z) = (x+y-z)(1-2x)$$

$$15. 45x^3y^4z^5 + 60x^5y^2z - 90x^4y^3z^2 = 15x^3y^2z(3y^2z^4 + 4x^2 - 6xy^3z)$$

$$16. 39a^5b^5c^3 - 65a^5b^3cd = 13a^5b^3c(3b^2c^2 - 5d)$$

$$17. -51a^4b^5c + 17a^3b^2c - 32a^3b^5c^4 = 17a^3b^2c(-3ab^3 + 1 - 2b^3c^3)$$

$$18. a(x-y) + b(y-x) = a(x-y) - b(x-y) \\ = (x-y)(a-b)$$

$$19. 2a(b-c) - 4c(c-b) = 2a(b-c) + 4c(b-c) \\ = (b-c)(2a+4c)$$

$$20. 9a^2b(x+y) + 3ab^2(x+y) = 3ab(x+y)(3a+b)$$

$$21. a(m-n) - b(n-m) = a(m-n) + b(m-n) \\ = (a+b)(m-n)$$

$$22. 5a^2(b-2) + 15a(2-b) = 5a^2(b-2) - 15a(b-2) \\ = 5a(b-2)(a-3)$$

Semaine 2 : Factoriser en utilisant les produits remarquables :

$$1. 16a^2 - 25y^2 = (4a - 5y)(4a + 5y)$$

$$2. x^2y^2z^2 - p^2 = (xyz - p)(xyz + p)$$

$$3. \frac{x^2}{4} - \frac{y^2}{25} = \left(\frac{x}{2} - \frac{y}{5}\right)\left(\frac{x}{2} + \frac{y}{5}\right)$$

$$4. a^3 - 4a^9 = 100a^2 + 9b^6 - 60ab^3 = (10a - 3b^3)^2$$

$$5. (a - 1)^2 - 1 = (a - 1 - 1)(a - 1 + 1) \\ = a(a - 2)$$

$$6. a^4 - 2a^2 + 1 = (a^2 - 1)^2 \\ = (a - 1)^2(a + 1)^2$$

$$7. 81a^4 - 169 = (9a^2 - 13)(9a^2 + 13)$$

$$8. \left(5x - \frac{3}{2}\right)^2 - \frac{81}{4} = \left(5x - \frac{3}{2} - \frac{9}{2}\right)\left(5x - \frac{3}{2} + \frac{9}{2}\right) \\ = (5x - 6)(5x + 3)$$

$$9. 25x^5y - 49xy^3 = 25a^2 - (b-2a)^2 = [5a - (b-2a)] \dots \\ \dots [5a + (b-2a)] \\ = (3a - b)(3a + b)$$

$$10. \frac{a^2}{4} + \frac{1}{49} - \frac{a}{7} = \left(\frac{a}{2} - \frac{1}{7} \right)^2$$

$$11. 81x^4 - \frac{1}{16} = \left(9x^2 - \frac{1}{4} \right) \left(9x^2 + \frac{1}{4} \right) \\ = \left(3x - \frac{1}{2} \right) \left(3x + \frac{1}{2} \right) \left(9x^2 + \frac{1}{4} \right)$$

$$12. (a-b)^2 - 121 = (a-b-11)(a-b+11)$$

$$13. a^4 - b^4 = (a^2 - b^2)(a^2 + b^2) \\ = (a-b)(a+b)(a^2 + b^2)$$

$$14. (2x+5)^2 - (3x-2)^2 = [(2x+5) - (3x-2)][(2x+5) + (3x-2)] \\ = (-x+7)(5x+3)$$

$$15. x^4 + 1 - 2x^2 = (x^2 - 1)^2 \\ = (x-1)^2(x+1)^2$$

$$16. \frac{xy}{3} + \frac{y^2}{9} + \frac{x^2}{4} = \left(\frac{x}{2} + \frac{y}{3} \right)^2$$

$$17. 49x^2 - (x-y)^2 = [7x - (x-y)][7x + (x-y)] \\ = (6x+y)(8x-y)$$

$$18. -x^2 + 12x - 36 \quad x^2 - 2 + \frac{1}{x^2} = \left(x - \frac{1}{x} \right)^2$$

Semaine 3 : Factoriser en utilisant les produits remarquables (une mise en évidence préalable peut parfois être nécessaire) :

$$1. 2a - 8a^3 + 8a^5 = 2a(1 - 4a^2 + 4a^4) \\ = 2a(1 - 2a^2)^2$$

$$2. 64x^5 + 16x^4 + x^3 = x^3(64x^2 + 16x + 1) \\ = x^3(8x + 1)^2$$

$$3. x^2 - 2 + \frac{1}{x^2} \quad a^3 - 4a^9 = a^3(1 - 4a^6) \\ = a^3(1 - 2a^3)(1 + 2a^3)$$

$$4. 100a^2 + 9b^6 - 30ab^3 \quad 25x^5y - 49xy^3 = xy(25x^4 - 49y^2) \\ = xy(5x^2 - 7y)(5x^2 + 7y)$$

$$5. 25x^4 + 16x^2y^4 - 40x^3y^2 = x^2(25x^2 + 16y^4 - 40xy^2) \\ = x^2(5x - 4y^2)^2$$

$$6. 4a^5 + 12a^3b^4 + 9ab^8 = a(4a^4 + 12a^2b^4 + 9b^8) \\ = a(2a^2 + 3b^4)^2$$

$$7. (2a + 1)^2 - (3 - a)^2 = [(2a + 1) - (3 - a)][(2a + 1) + (3 - a)] \\ = (3a - 2)(a + 4)$$

$$8. 9 + \frac{a^2b^2}{4} - 3ab = \left(3 - \frac{ab}{2}\right)^2$$

$$9. 25a^2 - (b - 2a)^2 = x^2 + 12x - 36 = -(x^2 - 12x + 36) \\ = -(x - 6)^2$$

Factoriser en utilisant les groupements :

$$\begin{aligned} 1. x^4 - 2x^3 + 4x - 8 &= (x^4 - 2x^3) + (4x - 8) \\ &= x^3(x - 2) + 4(x - 2) = (x - 2)(x^3 + 4) \end{aligned}$$

$$\begin{aligned} 2. 20a^3 + 5a^2 - 4a - 1 &= (20a^3 + 5a^2) - (4a + 1) \\ &= 5a^2(4a + 1) - (4a + 1) = (4a + 1)(5a^2 - 1) \end{aligned}$$

$$\begin{aligned} 3. 12ab - 8a + 3b - 2 &= (12ab - 8a) + (3b - 2) \\ &= 4a(3b - 2) + (3b - 2) = (4a + 1)(3b - 2) \end{aligned}$$

$$\begin{aligned} 4. a^3 - b^4 + a^2b - ab^3 &= (a^3 + a^2b) - (b^4 + ab^3) \\ &= a^2(a + b) - b^3(b + a) = (a + b)(a^2 - b^3) \end{aligned}$$

$$5. cy + y + c + 1 = (cy + y) + (c + 1)$$

$$= y(c + 1) + (c + 1) = (c + 1)(y + 1)$$

$$\begin{aligned} 6. 4x^2 + 2x - 9y^2 - 3y &= (4x^2 - 9y^2) + (2x - 3y) \\ &= (2x - 3y)(2x + 3y) + (2x - 3y) = (2x - 3y)(2x + 3y + 1) \end{aligned}$$

$$\begin{aligned} 7. c^2 + d - d^2 - c &= (c^2 - d^2) - (c - d) \\ &= (c - d)(c + d) - (c - d) = (c - d)(c + d - 1) \end{aligned}$$

$$\begin{aligned} 8. b^2y - b^2 + a^2y - a^2 &= (b^2y - b^2) + (a^2y - a^2) \\ &= b^2(y - 1) + a^2(y - 1) = (y - 1)(a^2 + b^2) \end{aligned}$$

$$\begin{aligned} 9. 5a^3 + a^2 - 20a - 4 &= (5a^3 + a^2) - (20a + 4) \\ &= a^2(5a + 1) - 4(5a + 1) \\ &= (5a + 1)(a^2 - 4) = (5a + 1)(a - 2)(a + 2) \end{aligned}$$

$$\begin{aligned} 10. a^4 - 2a^3 + a - 2 &= (a^4 - 2a^3) + (a - 2) \\ &= a^3(a - 2) + (a - 2) \\ &= (a - 2)(a^3 + 1) \end{aligned}$$

Semaine 4 : Factoriser en utilisant la méthode des diviseurs binômes :

1. $x^2 + 10x + 16$

div 16 : $\{\pm 1, \pm 2, \pm 4, \pm 8, \pm 16\}$

$P(1) \neq 0, P(-1) \neq 0, P(2) \neq 0, P(-2) = 0$

	1	10	16
-2		-2	-16
	1	8	0

$$x^2 + 10x + 16 = (x + 2)(x + 8)$$

2. $x^2 - 4x - 32$

div 32 : $\{\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32\}$

$P(-4) = 0$

	1	-4	-32
-4		-4	32
	1	-8	0

$$x^2 - 4x - 32 = (x + 4)(x - 8)$$

3. $x^2 + 5x - 14$

div 14: $\{ \pm 1, \pm 2, \pm 7, \pm 14 \}$

$P(x) = 0$

	1	5	-14
(2)		2	14
	1	7	0

$x^2 + 5x - 14 = (x - 2)(x + 7)$

4. $x^2 + 20x + 19$

div 19: $\{ \pm 1, \pm 19 \}$

$P(x) = 0$

	1	20	19
(-1)		-1	-19
	1	19	0

$x^2 + 20x + 19 = (x + 1)(x + 19)$

$$5. x^2 - 4x - 12$$

$$\text{div } 12: \{ \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12 \}$$

$$P(-2) = 0$$

	1	-4	-12
-2		-2	12
	1	-6	0

$$x^2 - 4x - 12 = (x + 2)(x - 6)$$

$$6. x^4 + 5x^3 - 15x^2 - 45x + 54$$

$$\text{div } 54: \{ \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm 27, \pm 54 \}$$

$$P(1) = 0, P(3) = 0, P(-3) = 0, P(-6) = 0$$

	1	5	-15	-45	54
1		1	6	-9	-54
	1	6	-9	-54	0
3		3	27	54	
	1	9	18	0	
-3		-3	-18		
	1	6	0		

$$x^4 + 5x^3 - 15x^2 - 45x + 54 = (x - 1)(x - 3)(x + 3)(x + 6)$$

$$7. x^4 + 2x^3 - 16x^2 - 2x + 15$$

$$\text{div } 15: \{ \pm 1, \pm 3, \pm 5, \pm 15 \}$$

$$P(1)=0, P(-1)=0, P(3)=0, P(-5)=0$$

	1	2	-16	-2	15
1		1	3	-13	-15
	1	3	-13	-15	0
-1		-1	-2	15	
	1	2	-15		
3		3	15		
	1	5	0		

$$x^4 + 2x^3 - 16x^2 - 2x + 15 = (x-1)(x+1)(x-3)(x+5)$$

$$8. x^4 - 7x^3 + 17x^2 - 17x + 6$$

$$\text{div } 6: \{ \pm 1, \pm 2, \pm 3, \pm 6 \}$$

$$P(1)=0, P(2)=0, P(3)=0$$

	1	-7	17	-17	6
1		1	-6	11	-6
	1	-6	11	-6	0
1		1	-5	6	
	1	-5	6	0	
2		2	-6		
	1	-3	0		

$$x^4 - 7x^3 + 17x^2 - 17x + 6 = (x-1)^2(x-2)(x-3)$$

9. $x^5 + 3x^4 - 16x - 48$

$P(48) = \{ \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16, \pm 24, \pm 48 \}$

$P(2) = 0, P(-2) = 0, P(-3) = 0$

	1	3	0	0	-16	-48
(2)		2	10	20	40	48
	1	5	10	20	24	0
(-2)		-2	-6	-8	-24	
	1	3	4	12	0	
(-3)		-3	0	-12		
	1	0	4	0		

$x^5 + 3x^4 - 16x - 48 = (x - 2)(x + 2)(x + 3)(x^2 + 4)$

Semaine 4 : Produits remarquables du 3^{ème} degré :

Développer

$$\begin{aligned} 1. (3a + b)^3 &= (3a)^3 + 3 \cdot (3a)^2 \cdot b + 3 \cdot 3a \cdot b^2 + b^3 \\ &= 27a^3 + 27a^2b + 9ab^2 + b^3 \end{aligned}$$

$$\begin{aligned} 2. (a - 5)^3 &= a^3 + 3a^2(-5) + 3a(-5)^2 + (-5)^3 \\ &= a^3 - 15a^2 + 75a - 125 \end{aligned}$$

$$\begin{aligned} 3. (2a + 3b)^3 &= (2a)^3 + 3(2a)^2(3b) + 3(2a)(3b)^2 + (3b)^3 \\ &= 8a^3 + 36a^2b + 54ab^2 + 27b^3 \end{aligned}$$

$$\begin{aligned} 4. (4 - 3a)^3 &= 4^3 - 3(4)^2(3a) + 3(4)(3a)^2 - (3a)^3 \\ &= 64 - 144a + 108a^2 - 27a^3 \end{aligned}$$

$$\begin{aligned} 5. \left(x + \frac{1}{x}\right)^3 &= x^3 + 3x^2 \frac{1}{x} + 3x \frac{1}{x^2} + \frac{1}{x^3} \\ &= x^3 + 3x + \frac{3}{x} + \frac{1}{x^3} \end{aligned}$$

$$6. \left(\frac{x}{2} + \frac{x^2}{3}\right)^3 = \left(\frac{x}{2}\right)^3 + 3\left(\frac{x}{2}\right)^2\left(\frac{x^2}{3}\right) + 3\left(\frac{x}{2}\right)\left(\frac{x^2}{3}\right)^2 + \left(\frac{x^2}{3}\right)^3$$

$$= \frac{x^3}{8} + \frac{x^4}{2} + \frac{x^5}{6} + \frac{x^6}{27}$$

$$7. \left(2x - \frac{3}{2}x\right)^3 = (2x)^3 - 3(2x)^2\left(\frac{3}{2}x\right) + 3(2x)\left(\frac{3}{2}x\right)^2 - \left(\frac{3}{2}x\right)^3$$

$$= 8x^3 - 18x^3 + 27x^3 - \frac{27}{8}x^3$$

$$= \frac{x^3}{8}$$

Factoriser

$$1. x^3 - 8y^3 = (x - 2y)(x^2 + 2xy + 4y^2)$$

$$2. 64a^3 + 125b^3 = (8a + 5b)(64a^2 - 40ab + 25b^2)$$

$$3. x^3 - \frac{27}{x^6} = \left(x - \frac{3}{x^2}\right)\left(x^2 + \frac{3}{x} + \frac{9}{x^4}\right)$$

$$4. \frac{a^3}{125} - 3\frac{a^2b}{50} + 3\frac{ab^2}{20} - \frac{b^3}{8} = \left(\frac{a}{5} - \frac{b}{2}\right)^3$$

$$5. \frac{8x^3}{27} + \frac{x^2y}{3} + \frac{xy^2}{8} + \frac{y^3}{64} = \left(\frac{2x}{3} + \frac{y}{4}\right)^3$$

$$6. 216a^6 + b^3 = (6a^2 + b)(36a^4 + 6a^2b + b^2)$$

$$7. \frac{64}{x^{12}} - 729x^6 = \left(\frac{4}{x^4} - 9x^2\right)\left(\frac{16}{x^8} + \frac{36}{x^2} + 81x^4\right)$$

$$= \left(\frac{2}{x^2} - 3x\right)\left(\frac{2}{x^2} + 3x\right)\left(\frac{16}{x^8} + \frac{36}{x^2} + 81x^4\right)$$

$$8. b^3 + 6b^2 + 12b + 8 = (b + 2)^3$$

$$9. 8a^3 - 60a + \frac{150}{a} - \frac{125}{a^3} = \left(2a - \frac{5}{a}\right)^3$$

Semaine 5 : Exercices mélangés :

$$1. 49x^3 - x = x(49x^2 - 1) = x(7x - 1)(7x + 1)$$

$$2. \frac{x^2}{4} + \frac{1}{49} - \frac{x}{7} = \left(\frac{x}{2} - \frac{1}{7}\right)^2$$

$$3. 9x^2y^2 - 16 = (3xy - 4)(3xy + 4)$$

$$4. 4a^2b^3 - b = b(4a^2b^2 - 1) = b(2ab - 1)(2ab + 1)$$

$$5. 75a^2 + 30a + 3 = 3(25a^2 + 10a + 1) = 3(5a + 1)^2$$

$$6. (x+3)^2 - 16 = [(x+3) - 4][(x+3) + 4] \\ = (x-1)(x+7)$$

$$7. 4x^2(x+3) - (3+x) = (3+x)(4x^2 - 1) = (3+x)(2x-1)(2x+1)$$

$$8. 9(x^2 - 4) - 4(x^2 - 4) = (x^2 - 4)(9 - 4) \\ = (x-2)(x+2)(3-y)(3+y)$$

$$9. 3x(y^2 - 2y + 1) + 3x = 3x(y^2 - 2y + 1) \\ = 3x(y-1)^2$$

$$10. x^4 - 2x^2 + 1 = (x^2 - 1)^2 = (x - 1)^2 (x + 1)^2$$

$$11. (4x^2 + 12x)^2 + 9(4x^2 + 12x) = (4x^2 + 12x)(4x^2 + 12x + 9) \\ = 4x(x + 3)(2x + 3)^2$$

$$12. 3x^4 \left(x^2 - \frac{4}{9}\right) - 75 \left(x^2 - \frac{4}{9}\right) = 3 \left(x^2 - \frac{4}{9}\right) (x^4 - 25) \\ = 3 \left(x - \frac{2}{3}\right) \left(x + \frac{2}{3}\right) (x^2 - 5) (x^2 + 5)$$

$$13. 2a(x + y) - 3b(-x - y) = 2a(x + y) + 3b(x + y) \\ = (x + y)(2a + 3b)$$

$$14. x(2a - b) + y(b - 2a) = x(2a - b) - y(2a - b) \\ = (2a - b)(x - y)$$

$$15. a(x - y) - (y - x) = a(x - y) + 1(x - y) \\ = (x - y)(a + 1)$$

$$16. (4a - 2b)(2x - 3y) + (3y - 2x)(b - 2a) = 2(2a - b)(2x - 3y) + (2x - 3y)(2a - b) \\ = 3(2x - 3y)(2a - b)$$

$$17. a^2(x - 1)(a + b) + a^3(1 - x) = a^2(x - 1)(a + b) - a^3(x - 1) \\ = a^2(x - 1)[(a + b) - a] \\ = a^2b(x - 1)$$

$$18. (x - 2y)(a - b) - (b - a)(2x + y) = (x - 2y)(a - b) + (a - b)(2x + y) \\ = (a - b)[(x - 2y) + (2x + y)] \\ = (a - b)(3x - y)$$

$$19. 5x^2 - 5 = 5(x^2 - 1) = 5(x-1)(x+1)$$

$$20. a^4x^4 - a^4 = a^4(x^4 - 1) = a^4(x^2 - 1)(x^2 + 1) \\ = a^4(x-1)(x+1)(x^2 + 1)$$

$$21. a^4b^2x^5 - a^2x = a^2x(a^2b^2x^4 - 1) \\ = a^2x(abx^2 - 1)(abx^2 + 1)$$

$$22. 9x^5y^7 - xy = xy(9x^4y^6 - 1) = xy(3x^2y^3 - 1)(3x^2y^3 + 1)$$

$$23. 12(x+2)^3 - 3(x+2) = 3(x+2)[4(x+2)^2 - 1] \\ = 3(x+2)[2(x+2) - 1][2(x+2) + 1] = \textcircled{\times}$$

$$24. (a-b) - (a-b)x^4 = (a-b)(1-x^4) = (a-b)(1-x^2)(1+x^2) \\ = (a-b)(1-x)(x+1)(x^2+1)$$

$$25. -3x^9 + 3x^5 = -3x^5(x^4 - 1) \\ = -3x^5(x^2 - 1)(x^2 + 1) \\ = -3x^5(x-1)(x+1)(x^2+1)$$

$$\textcircled{\times} = 3(x+2)(2x+3)(2x+5)$$

$$26. 24x - 4 + 5x^4 - 6x^3 - 19x^2 = 5x^4 - 6x^3 - 19x^2 + 24x - 4$$

$$\text{div } 4 = \{ \pm 1, \pm 2, \pm 4 \}$$

$$P(1) = 0, P(2) = 0, P(-2) = 0$$

	5	-6	-19	24	-4
(1)		5	-1	-20	4
	5	-1	-20	4	0
(2)		10	18	-4	
	5	9	-2	0	
(-2)		-10	2		
	5	-1	0		

$$5x^4 - 6x^3 - 19x^2 + 24x - 4 = (x-1)(x-2)(x+2)(5x+1)$$