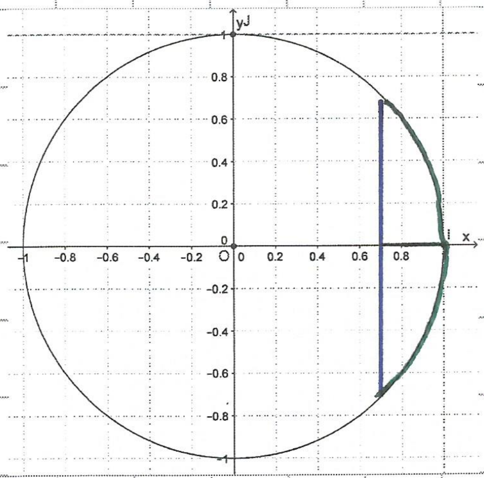
Inéquations trigonométriques : Solutions

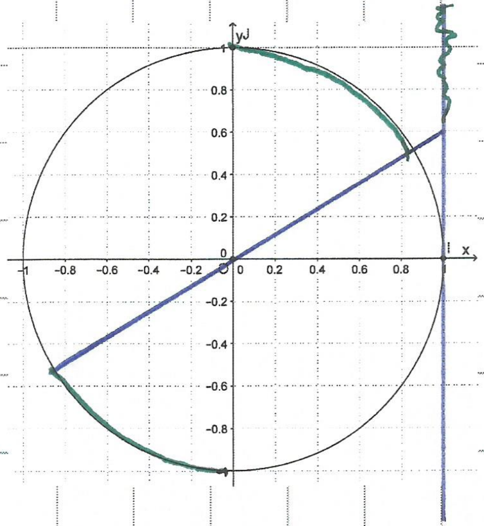
1. $\cos x > \dfrac{\sqrt{3}}{2}$



$S : \left]-\frac{\pi}{6}, \frac{\pi}{6}\right[+ 2k\pi$

ou $S : \left]-\frac{\pi}{6}, \frac{\pi}{6}\right[$ sur $]-\pi, \pi]$

2. $3\tan y - \sqrt{3} \geq 0$ $\iff \tan y \geq \frac{\sqrt{3}}{3}$



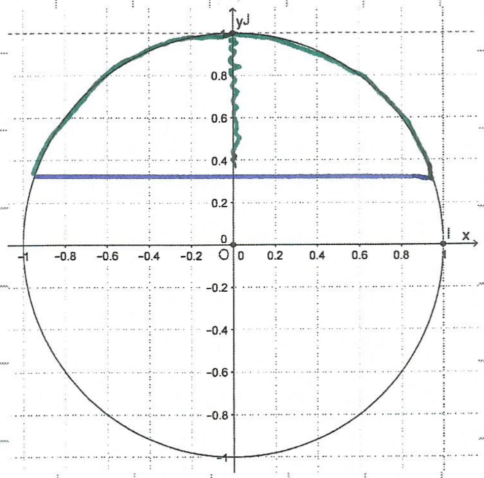
$S : \left[\frac{\pi}{6}, \frac{\pi}{2}\right[+ k\pi$

ou $S : \left[-\frac{5\pi}{6}, -\frac{\pi}{2}\right[\cup$

$\left[\frac{\pi}{6}, \frac{\pi}{2}\right[$ sur $]-\pi, \pi]$

3. $1 - 3\sin x \leq 0$

$\sin x \geq \frac{1}{3}$



$S : [0{,}34 \, ; 2{,}80] + 2k\pi$

ou $S : [0{,}34 \, ; 2{,}80]$ sur $]-\pi, \pi]$

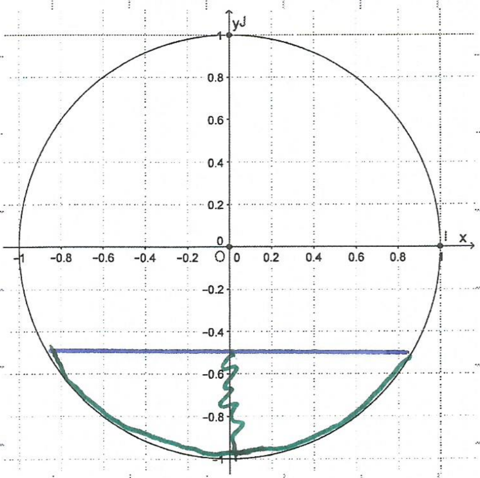
4. $2\sin 3t + 1 < 0$

$\iff \sin 3t < -\frac{1}{2}$

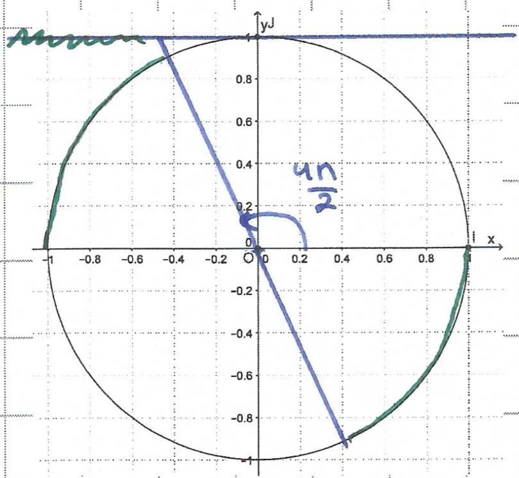
$\iff 3t \in \left]\frac{7\pi}{6}, \frac{11\pi}{6}\right[+ 2k\pi$

$\iff t \in \left]\frac{7\pi}{18}, \frac{11\pi}{18}\right[+ \frac{2k\pi}{3}$

ou $S: \left]-\frac{17\pi}{18}, -\frac{13\pi}{18}\right[\cup \left]-\frac{5\pi}{18}, -\frac{\pi}{18}\right[\cup \left]\frac{7\pi}{18}, \frac{11\pi}{18}\right[$ sur $]-\pi, \pi]$



5. $\cot\frac{4\pi}{7} - \cot 3x \geq 0$ $\iff \cot 3x \leq \cot\frac{4\pi}{7}$



$\iff 3x \in \left[\frac{4\pi}{7}, \pi\right[+ k\pi$

$\iff x \in \left[\frac{4\pi}{21}, \frac{\pi}{3}\right[+ \frac{k\pi}{3}$

on S: $\left[-\frac{17\pi}{21}, -\frac{2\pi}{3}\right[\cup \left[-\frac{10\pi}{21}, -\frac{\pi}{3}\right[$

$\cup \left[-\frac{3\pi}{21}, 0\right[\cup \left[\frac{4\pi}{21}, \frac{\pi}{3}\right[$

$\cup \left[\frac{11\pi}{21}, \frac{2\pi}{3}\right[\cup \left[\frac{18\pi}{21}, \pi\right[$ sur $]-\pi, \pi]$

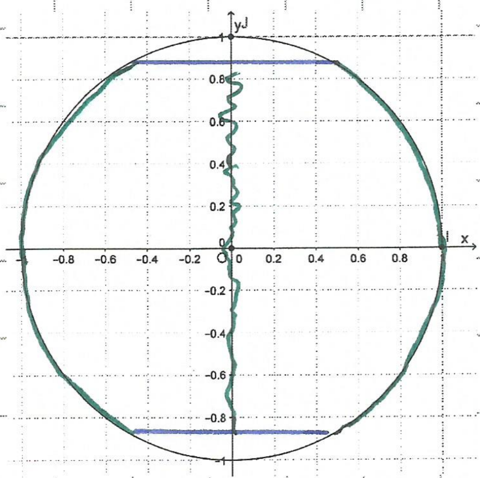
6. $\sin^2 3x \leq \dfrac{3}{4}$

$\sin 3x$		$-\frac{\sqrt{3}}{2}$		$\frac{\sqrt{3}}{2}$	
$\sin^2 3x - \frac{3}{4}$	$+$	0	$-$	0	$+$

$\iff \sin 3x \in \left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$

$S: 3x \in \left[0, \frac{\pi}{3}\right] \cup \left[\frac{2\pi}{3}, \frac{4\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 2\pi\right[+ 2k\pi$

$\iff x \in \left[0, \frac{\pi}{9}\right] \cup \left[\frac{2\pi}{9}, \frac{4\pi}{9}\right] \cup \left[\frac{5\pi}{9}, \frac{2\pi}{3}\right[+ \frac{2k\pi}{3}$



ou $S: \left]-\pi, -\frac{8\pi}{9}\right] \cup \left[-\frac{7\pi}{9}, -\frac{5\pi}{9}\right]$

$\cup \left[-\frac{4\pi}{9}, -\frac{2\pi}{9}\right] \cup \left[-\frac{\pi}{9}, \frac{\pi}{9}\right]$

$\cup \left[\frac{2\pi}{9}, \frac{4\pi}{9}\right] \cup \left[\frac{5\pi}{9}, \frac{7\pi}{9}\right]$

$\cup \left[\frac{8\pi}{9}, \pi\right]$ sur $\left]-\pi, \pi\right]$

7. $2\sin^2 x - 3\sin x + 1 < 0$

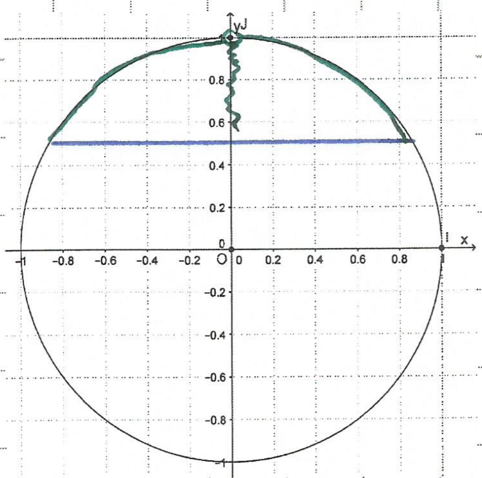
$\Delta = 9 - 8 = 1$

$\sin x_{1,2} = \dfrac{3 \pm 1}{4} \begin{cases} 1 \\ \frac{1}{2} \end{cases}$

$\sin x$		$\frac{1}{2}$		1	
I_x	$+$	0	$-$	0	$+$

$\sin x \in \left] \frac{1}{2}, 1 \right[$

$x \in \left] \frac{\pi}{6}, \frac{\pi}{2} \right[\cup \left] \frac{\pi}{2}, \frac{5\pi}{6} \right[$ sur $]-\pi, \pi]$



8. $\dfrac{2\sin 2x - 1}{\cos 2x - 3\cos x + 2} > 0$

Pour faire le tableau de signe, on cherche les zéros

N : $2\sin 2x - 1 = 0 \iff \sin 2x = \frac{1}{2}$

$$\iff \begin{cases} 2x = \frac{\pi}{6} + 2k\pi \\ 2x = \frac{5\pi}{6} + 2k\pi \end{cases} \iff \begin{cases} x = \frac{\pi}{12} + k\pi \\ x = \frac{5\pi}{12} + k\pi \end{cases}$$

Sur $]-\pi, \pi]$, les solutions sont : $\left\{-\frac{11\pi}{12}, -\frac{7\pi}{12}, \frac{\pi}{12}, \frac{11\pi}{12}\right\}$

D : $\cos 2x - 3\cos x + 2 = 0 \iff 2\cos^2 x - 1 - 3\cos x + 2 = 0$

$\iff 2\cos^2 x - 3\cos x + 1 = 0$

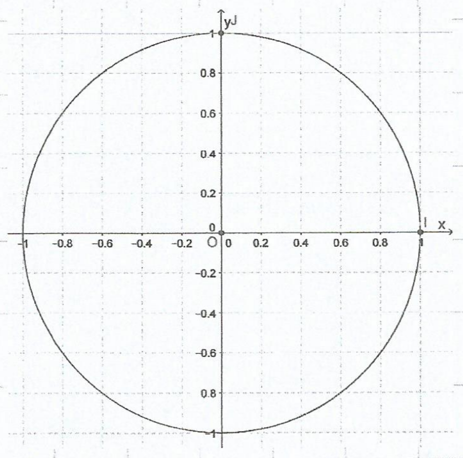
$\Delta = 1$, $\cos x_{1,2} = \frac{3 \pm 1}{4} < \begin{matrix} 1 \\ \frac{1}{2} \end{matrix}$

$\cos x = 1 \iff x = 2k\pi$

$\cos x = \frac{1}{2} \iff x = \pm\frac{\pi}{3} + 2k\pi$

Rmq : D se factorise en

$$2\left(\cos x - \frac{1}{2}\right)(\cos x - 1)$$



x	$-\pi$		$-\frac{11\pi}{12}$		$-\frac{7\pi}{12}$		$-\frac{\pi}{3}$		0		$\frac{\pi}{12}$		$\frac{\pi}{3}$		$\frac{5\pi}{12}$		π	
N		$-$	0	$+$	0	$-$		$-$		$-$	0	$+$		$+$	0	$-$		(1)
D $\cos x - \frac{1}{2}$		$-$		$-$		$-$	0	$+$		$+$		$+$	0	$-$		$-$		
D $\cos x - 1$		$-$		$-$		$-$		$-$	0	$-$		$-$		$-$		$-$		(2)
		$-$	0	$+$	0	$-$	$\Vert$	$+$	$\Vert$	$+$	0	$-$	0	$+$	0	$-$		

$S : \left]-\frac{11\pi}{12}, -\frac{7\pi}{12}\right[\cup \left]-\frac{\pi}{3}, 0\right[\cup \left]0, \frac{\pi}{12}\right[\cup \left]\frac{\pi}{3}, \frac{5\pi}{12}\right[$

(1) On remplace x par 0 et on voit le signe de $2\sin 2x - 1$ (ici <0). Chaque fois que l'on rencontre un 0, on change de signe

(2) C'est un cas particulier où la fct ne change pas de signe même si elle s'annule

$$-1 \leq \cos x \leq 1$$

$$\Longleftrightarrow -2 \leq \cos x - 1 \leq 0$$

⚠ cet exercice est difficile et demande de la rigueur !!

9. $\tan^2 2x + (1-\sqrt{3})\tan 2x - \sqrt{3} \leq 0$

$\Delta = 1 + 3 - 2\sqrt{3} + 4\sqrt{3} = (1+\sqrt{3})^2$

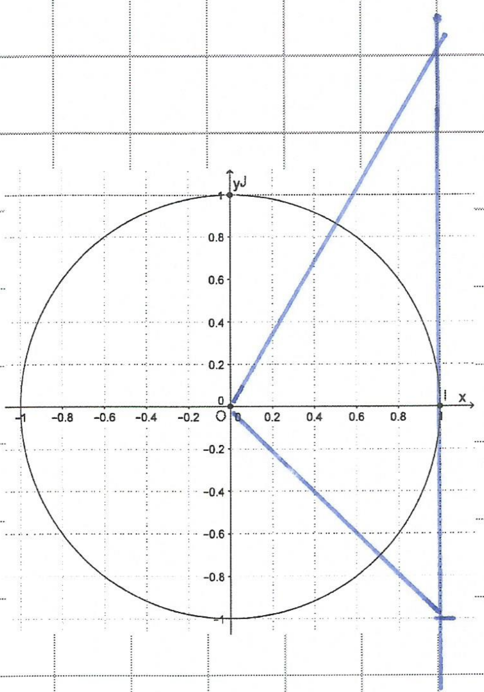
$\tan 2x_{1,2} = \underline{-1+\sqrt{3} \pm (1+\sqrt{3})}$ $\begin{cases} \sqrt{3} \\ -1 \end{cases}$

$\tan 2x$		-1		$\sqrt{3}$	
I_n	$+$	0	$-$	0	$+$

$\tan 2x \in [-1, \sqrt{3}]$

$2x \in \left[-\frac{\pi}{4}, \frac{\pi}{3}\right] + k\pi$

$x \in \left[-\frac{\pi}{8}, \frac{\pi}{6}\right] + \frac{k\pi}{2}$



ou $S : \left]-\pi, \frac{5\pi}{6}\right] \cup \left[-\frac{5\pi}{8}, -\frac{\pi}{3}\right]$

$\cup \left[-\frac{\pi}{8}, \frac{\pi}{6}\right] \cup \left[\frac{3\pi}{8}, \frac{2\pi}{3}\right]$

$\cup \left[\frac{7\pi}{8}, \pi\right]$ sur $]-\pi, \pi]$

10. $\sin 3x - \sin 2x + \sin x \geq 0 \iff 2 \sin 2x \cos x - \sin 2x \geq 0$

(Simpson avec (1) et (3))

$\iff \sin 2x (2 \cos x - 1) \geq 0$

zéros : $\sin 2x = 0 \iff 2x = k\pi \iff x = \frac{k\pi}{2}$

$2\cos x - 1 = 0 \iff \cos x = \frac{1}{2} \iff x = \pm \frac{\pi}{3} + 2k\pi$

x	$-\pi$		$-\frac{\pi}{2}$		$-\frac{\pi}{3}$		0		$\frac{\pi}{3}$		$\frac{\pi}{2}$		π
$\sin 2x$	0	+	0	−		−	0	+		+	0	−	0
$2\cos x - 1$		−		−	0	+		+	0	−		−	
	0	−	0	+	0	−	0	+	0	−	0	+	0

S : $[-\frac{\pi}{2}, -\frac{\pi}{3}] \cup [0, \frac{\pi}{3}] \cup [\frac{\pi}{2}, \pi]$