Exercices complémentaires limites : Solutions

Calculer les limites suivantes :

1. $\lim\limits_{x\to\pm\infty} \dfrac{\sqrt{x^2-1}}{2\sqrt[3]{2x^3-x}} = \dfrac{\infty}{\infty}$ F.I.

$$= \lim_{\pm\infty} \frac{\sqrt{x^2}}{2\sqrt[3]{2x^3}} = \lim_{\pm\infty} \frac{|x|}{2\sqrt[3]{2}\,.\,x}$$

$$= \begin{cases} \lim\limits_{-\infty} \dfrac{-\cancel{x}}{2\sqrt[3]{2}\,\cancel{x}} = -\dfrac{1}{2\sqrt[3]{2}} \cdot \dfrac{\sqrt[3]{4}}{\sqrt[3]{4}} = \dfrac{-\sqrt[3]{4}}{4} \\ \lim\limits_{+\infty} \dfrac{\cancel{x}}{2\sqrt[3]{2}\,\cancel{x}} = \dfrac{1}{2\sqrt[3]{2}} = \dfrac{\sqrt[3]{4}}{4} \end{cases}$$

2. $\lim_{x \to 5} \dfrac{x^3 - 125}{x^2 - 6x + 5} = \dfrac{0}{0}$ FI

$$\stackrel{\Delta}{=} \lim_{5} \frac{\cancel{(x-5)}(x^2+5x+25)}{\cancel{(x-5)}(x-1)}$$

$$= \frac{75}{4}$$

3. $\lim\limits_{x\to 1} \dfrac{x^2 + \dfrac{x+3}{x-1}}{x+2-\dfrac{x^2}{x-1}} = \left(\dfrac{\infty}{\infty}\right)$

$$\lim_{1} \frac{\dfrac{x^3 - x^2 + x + 3}{\cancel{x-1}}}{\dfrac{(x+2)(x-1) - x^2}{\cancel{x-1}}}$$

$\square$: on peut simplifier car lim !

$$= \lim_{1} \frac{x^3 - x^2 + x + 3}{\cancel{x^2} + x - 2 - \cancel{x^2}}$$

$$= \frac{4}{-1} = -4$$

4. $\lim_{x\to\pm\infty}\left[\sqrt{16x^2+3x-5}-4x-2\right]=$

$$\cdot \lim_{-\infty}[\quad] = \infty + \infty = \infty$$

$$\cdot \lim_{+\infty}[\quad] = \infty - \infty \quad \text{FI}$$

$$= \lim_{+\infty} \frac{\left[\sqrt{16x^2+3x-5}-(4x+2)\right]\left[\sqrt{\quad}+(4x+2)\right]}{\sqrt{\quad}+(4x+2)}$$

$$= \lim_{+\infty} \frac{16x^2+3x-5-(4x+2)^2}{D}$$

$$= \lim_{+\infty} \frac{\cancel{16x^2}+3x-5-(\cancel{16x^2}+16x+4)}{D}$$

$$= \lim_{+\infty} \frac{-13x-9}{\sqrt{16x^2+3x-5}+(4x+2)}$$

$$= \frac{\infty}{\infty} \quad \text{FI}$$

$$= \lim_{+\infty} \frac{-13x}{\sqrt{16x^2}+4x}$$

$$= \lim_{+\infty} \frac{-13x}{4x+4x}$$

$$= \lim_{+\infty} \frac{-13\cancel{x}}{8\cancel{x}}$$

$$= -\frac{13}{8}$$

5. $\lim\limits_{x \to -3} \dfrac{\sqrt{x^2-1} - \sqrt{x+11}}{\sqrt{x+6} - \sqrt{x^2-6}} = \dfrac{0}{0}$ FI

$$= \lim_{-3} \frac{\sqrt{x^2-1} - \sqrt{x+11}}{\sqrt{x+6} - \sqrt{x^2-6}} \cdot \frac{\sqrt{x^2-1} + \sqrt{x+11}}{\sqrt{x^2-1} + \sqrt{x+11}} \cdot \frac{\sqrt{x+6} + \sqrt{x^2-6}}{\sqrt{x+6} + \sqrt{x^2-6}}$$

$$= \lim_{-3} \frac{x^2 - 1 - (x+11)}{x+6 - (x^2-6)} \cdot \frac{\sqrt{x+6} + \sqrt{x^2-6}}{\sqrt{x^2-1} + \sqrt{x+11}}$$

$$= \lim_{-3} \frac{\cancel{x^2 - x - 12}^{\,-1}}{\cancel{-x^2 + x + 12}} \cdot \frac{\sqrt{x+6} + \sqrt{x^2-6}}{\sqrt{x^2-1} + \sqrt{x+11}}$$

$$= -1 \cdot \frac{\sqrt{3} + \sqrt{3}}{\sqrt{8} + \sqrt{8}}$$

$$= -\frac{\cancel{2}\sqrt{3}}{\cancel{2}\underbrace{\sqrt{8}}_{2\sqrt{2}}}$$

$$= -\frac{\sqrt{6}}{4}$$

6. $\lim\limits_{x\to 3} \dfrac{\sqrt[3]{7x+6}-3}{x-3} = \dfrac{0}{0}$ FI

$$= \lim_{3} \frac{\sqrt[3]{7x+6}-3}{x-3} \cdot \frac{\left(\sqrt[3]{7x+6}\right)^2 + 3\sqrt[3]{7x+6} + 9}{\left(\sqrt[3]{7x+6}\right)^2 + 3\sqrt[3]{7x+6} + 9}$$

$$= \lim_{3} \frac{\overbrace{7x+6-27}^{7\ \cancel{7x-21}}}{\left[\left(\sqrt[3]{\ }\right)^2 + 3\sqrt[3]{\ } + 9\right]\cancel{(x-3)}}$$

$$= \frac{7}{\sqrt[3]{27}^{\,2} + 3\sqrt[3]{27} + 9}$$

$$= \frac{7}{27}$$

7. $\lim_{x\to\pm\infty} \frac{x^2+x+1}{-3x^2+5x-2} = \frac{\infty}{\infty}$ FI

$= \lim_{\pm\infty} \frac{\cancel{x^2}}{-3\cancel{x^2}} = -\frac{1}{3}$

8. $\lim\limits_{x\to\pm\infty}\left[3x-1-\sqrt{9x^{\cancel{3}\,2}+2x-1}\right]=$

$\bullet \lim\limits_{-\infty}[\quad] = -\infty - \infty = -\infty$

$\bullet \lim\limits_{+\infty}[\quad] = \infty - \infty \quad \text{FI}$

$$= \lim_{+\infty} \frac{[(3x-1)-\sqrt{\quad}][(3x-1)+\sqrt{\quad}]}{(3x-1)+\sqrt{\quad}}$$

$$= \lim_{+\infty} \frac{(3x-1)^2-(9x^2+2x-1)}{(3x-1)+\sqrt{\quad}}$$

$$= \lim_{+\infty} \frac{\cancel{9x^2}-6x+1-\cancel{9x^2}-2x+1}{D}$$

$$= \lim_{+\infty} \frac{-8x+2}{D}$$

$$= \frac{\infty}{\infty} \quad \text{FI}$$

$$= \lim_{+\infty} \frac{-8x}{3x+\sqrt{9x^2}}$$

$$= \lim_{+\infty} \frac{-8x}{6x}$$

$$= -\frac{4}{3}$$

9. $\lim\limits_{x\to\pm\infty}\left[\sqrt{x^2+3x-5}-\sqrt{x^2-3x+2}\right] = \infty - \infty$ FI

$$= \lim_{\pm\infty} \frac{\left[\sqrt{\boxed{\ }} - \sqrt{\bigcirc}\right]\left[\sqrt{\boxed{\ }} + \sqrt{\bigcirc}\right]}{\sqrt{\boxed{\ }} + \sqrt{\bigcirc}}$$

$$= \lim_{\pm\infty} \frac{x^2+3x-5-(x^2-3x+2)}{\sqrt{\ } + \sqrt{\ }}$$

$$= \lim_{\pm\infty} \frac{6x-7}{D}$$

$$= \frac{\infty}{\infty} \quad \text{FI}$$

$$= \lim_{\pm\infty} \frac{6x}{\sqrt{x^2}+\sqrt{x^2}} = \lim_{\pm\infty} \frac{\overset{3}{\cancel{6}}x}{\cancel{2}|x|}$$

$$= \begin{cases} -\infty: & \lim\limits_{-\infty} \dfrac{3x}{-x} = -3 \\ +\infty: & \lim\limits_{+\infty} \dfrac{3x}{x} = 3 \end{cases}$$

10. $\lim\limits_{x\to1} \dfrac{\sqrt{x+3}-2}{2-\sqrt{3x+1}} = \dfrac{0}{0}$ FI

$$= \lim_{1} \frac{\sqrt{x+3}-2}{2-\sqrt{3x+1}} \quad \frac{\sqrt{x+3}+2}{\sqrt{x+3}+2} \quad \frac{2+\sqrt{3x+1}}{2+\sqrt{3x+1}}$$

$$= \lim_{1} \frac{x+3-4}{2-(3x-1)} \cdot \frac{2+\sqrt{3x+1}}{\sqrt{x+3}+2}$$

$$= \lim_{1} \frac{\cancel{x-1}}{\underset{-3}{\cancel{3-3x}}} \cdot \frac{2+\sqrt{3x+1}}{\sqrt{x+3}+2}$$

$$= -\frac{1}{3} \quad \frac{2+\sqrt{4}}{\sqrt{4}+2}$$

$$= -\frac{1}{3}$$