Chapitre 1

Trigonométrie

1. Montrer que

$$\cos^2 x + \cos^2(120° + x) + \cos^2(120° - x)$$

est indépendant de x.

$\cos^2 x + [\cos 120 \cos x - \sin 120 \sin x]^2 + \dots$

$\dots + [\cos 120 \cos x + \sin 120 \sin x$

$= \cos^2 x + \cos^2 120 \cos^2 x + \sin^2 120 \sin^2 x + \dots$

$+ \cos^2 120 \cos^2 x + \sin^2 120 \sin^2 x$

(les doubles produits disparaissent)

$= \cos^2 x + \left(\frac{1}{4} \cos^2 x + \frac{3}{4} \sin^2 x\right) 2$

$= \frac{3}{2} \cos^2 x + \frac{3}{2} \sin^2 x$

$= \frac{3}{2}$

2. Vérifier que $\tan^2 a - \tan^2 b = \dfrac{\sin(a+b)\sin(a-b)}{\cos^2 a \cos^2 b}$

$$\mathrm{II} = \frac{(\sin a \cos b + \sin b \cos a)(\sin a \cos b - \sin b \cos a)}{\cos^2 a \cos^2 b}$$

$$= \frac{\sin^2 a \cos^2 b - \sin^2 b \cos^2 a}{\cos^2 a \cos^2 b}$$

$$= \frac{\sin^2 a \cancel{\cos^2 b}}{\cos^2 a \cancel{\cos^2 b}} - \frac{\sin^2 b \cancel{\cos^2 a}}{\cancel{\cos^2 a} \cos^2 b}$$

$$= \tan^2 a - \tan^2 b$$

$$= \mathrm{I}$$

3. Vérifier les identités suivantes :

(a) $\dfrac{\cos(a+b)+\cos(a-b)}{\sin(a+b)+\sin(a-b)} = \cot a$

$$I = \frac{\cancel{2} \cos \frac{a+b+a-b}{2} \cancel{\cos \frac{a+b-(a-b)}{2}}}{\cancel{2} \sin \frac{a+b+a-b}{2} \cancel{\cos \frac{a+b-(a-b)}{2}}}$$

$$= \frac{\cos a}{\sin a}$$

$$= \cot a$$

$$= II$$

(b) $\cot a \cos 2a - \sin 2a = \dfrac{\cos 3a}{\sin a}$

$$\Longleftrightarrow \frac{\cos a}{\sin a} \cos 2a - \sin 2a = \text{II}$$

$$\Longleftrightarrow \frac{\cos a \cos 2a - \sin a \sin 2a}{\sin a} = \text{II}$$

$$\Longleftrightarrow \frac{\cos(a + 2a)}{\sin a} = \text{II}$$

$$\Longleftrightarrow \frac{\cos 3a}{\sin a} = \text{II}$$

(c) $\sin^2(a+b) + \cos^2(a-b) = 1 + \sin 2a \sin 2b$

$\Leftrightarrow (\sin a \cos b + \sin b \cos a)^2 + (\cos a \cos b + \sin a \sin b)^2 = \mathrm{II}$

$\Leftrightarrow \sin^2 a \cos^2 b + \sin^2 b \cos^2 a + 2 \sin a \cos a \sin b \cos b \ldots$

$\ldots + \cos^2 a \cos^2 b + \sin^2 a \sin^2 b + 2 \sin a \cos a \sin b \cos b = \mathrm{II}$

$\Leftrightarrow \sin^2 a \cos^2 b + \cos^2 a \cos^2 b + \sin^2 b \cos^2 a + \sin^2 a \sin^2 b \ldots$

$\ldots + 4 \sin a \cos a \sin b \cos b = \mathrm{II}$

$\Leftrightarrow \cos^2 b \,(\underbrace{\sin^2 a + \cos^2 a}_{1}) + \sin^2 b \,(\underbrace{\cos^2 a + \sin^2 a}_{1}) \ldots$

$\ldots + \overbrace{4}^{2\cdot 2} \underbrace{\sin a \cos a}_{\sin 2a} \underbrace{\sin b \cos b}_{\sin 2b} = \mathrm{II}$

$\Leftrightarrow \underbrace{\cos^2 b + \sin^2 b}_{1} + \sin 2a \sin 2b = \mathrm{II}$

$\Leftrightarrow 1 + \sin 2a \sin 2b = \mathrm{II}$

4. On donne $\tan a = \frac{4}{3}$ $(a \in \left]\pi, \frac{3\pi}{2}\right[)$
   (a) Calculer $\sin 2a$, $\cos 2a$ et $\tan 2a$
   (b) Dans quel quadrant se trouve l'angle $2a$? Justifier.

Si $\tan a = \frac{4}{3} \Rightarrow \frac{1}{\cos^2 a} = 1 + \frac{16}{9}$

$\Leftrightarrow \frac{1}{\cos^2 a} = \frac{25}{9} \Leftrightarrow \cos^2 a = \frac{9}{25} \Leftrightarrow \cos a = \cancel{+} - \frac{3}{5}$

$\hookrightarrow a \in Q_{III}$

et $\sin a = \tan \cos a \Leftrightarrow \sin a = -\frac{4}{5}$

a) $\sin 2a = 2 \sin a \cos a$

$= 2\left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right)$

$= \frac{24}{25}$

$\cos 2a = 2\cos^2 a - 1$

$= 2\,\frac{9}{25} - 1$

$= -\frac{7}{25}$

$\tan 2a = \frac{\sin 2a}{\cos 2a}$

$= -\frac{24}{7}$

b) $2a \in Q_{II}$ car $\cos 2a < 0$ et $\sin 2a > 0$