

Equations trigonométriques : Exercices complémentaires - Solutions

Résoudre les équations suivantes. Exprimer les solutions dans l'intervalle $] -\pi, \pi]$:

1. $2 \sin 2x + 1 = 0$

$$\Leftrightarrow \sin 2x = -\frac{1}{2}$$

$$\Leftrightarrow \begin{cases} 2x = -\frac{\pi}{6} + 2k\pi \\ 2x = -\frac{5\pi}{6} + 2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -\frac{\pi}{12} + k\pi \\ x = -\frac{5\pi}{12} + k\pi \end{cases}$$

$$\text{sur }]-\pi, \pi]: S: \left\{ -\frac{5\pi}{12}, -\frac{\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12} \right\}$$

2. $2 \cos 5x = -\sqrt{3}$

$$\cos 5x = -\frac{\sqrt{3}}{2} \Leftrightarrow \begin{cases} 5x = \frac{2\pi}{3} + 2k\pi \\ 5x = -\frac{2\pi}{3} + 2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} x = \frac{2\pi}{15} + \frac{2k\pi}{5} \\ x = -\frac{2\pi}{15} + \frac{2k\pi}{5} \end{cases}$$

$$\text{Sur }]-\pi, \pi]:$$

$$S: \left\{ -\frac{14\pi}{15}, -\frac{2\pi}{3}, -\frac{8\pi}{15}, -\frac{4\pi}{15}, -\frac{2\pi}{15}, \frac{2\pi}{15}, \frac{4\pi}{15}, \frac{8\pi}{15}, \frac{2\pi}{3}, \frac{14\pi}{15} \right\}$$

$$3. \ 2 \sin\left(\frac{\pi}{6} - 2x\right) = \sqrt{3}$$

$$\Leftrightarrow \sin\left(\frac{\pi}{6} - 2x\right) = \frac{\sqrt{3}}{2} \Leftrightarrow \begin{cases} \frac{\pi}{6} - 2x = \frac{\pi}{3} + 2k\pi & (*) \\ \frac{\pi}{6} - 2x = \frac{2\pi}{3} + 2k\pi & (n-x) \end{cases}$$

$$\Leftrightarrow \begin{cases} -2x = \frac{\pi}{6} + 2k\pi \\ -2x = \frac{\pi}{2} + 2k\pi \end{cases} \Leftrightarrow \begin{cases} x = -\frac{\pi}{12} + k\pi \\ x = -\frac{\pi}{4} + k\pi \end{cases}$$

$$S_{\sin]-\pi, \pi]} : S : \left\{ -\frac{\pi}{4}, -\frac{\pi}{12}, \frac{3\pi}{4}, \frac{11\pi}{12} \right\}$$

$$4. \ 3 \tan\left(3x - \frac{\pi}{4}\right) = -\sqrt{3}$$

$$\Leftrightarrow \tan\left(3x - \frac{\pi}{4}\right) = -\frac{\sqrt{3}}{3} \quad \underline{\Leftrightarrow} \quad \begin{aligned} 3x - \frac{\pi}{4} &= \frac{\pi}{2} + k\pi \\ 3x &\neq \frac{3\pi}{4} + k\pi \\ x &\neq \frac{3\pi}{12} + \frac{k\pi}{3} \end{aligned}$$

$$\Leftrightarrow 3x - \frac{\pi}{4} = -\frac{\pi}{6} + k\pi$$

$$\Leftrightarrow 3x = \frac{\pi}{12} + k\pi \Leftrightarrow x = \frac{\pi}{36} + \frac{k\pi}{3}$$

$$S_{\sin]-\pi, \pi]} :$$

$$S : \left\{ -\frac{35\pi}{36}, -\frac{23\pi}{36}, -\frac{11\pi}{36}, \frac{\pi}{36}, \frac{13\pi}{36}, \frac{25\pi}{36} \right\}$$

$$5. \sin 3x = \cos\left(\frac{\pi}{3} - x\right) \Leftrightarrow \sin 3x = \sin\left[\frac{\pi}{2} - \left(\frac{\pi}{3} - x\right)\right]$$

$$\Leftrightarrow \sin 3x = \sin\left(\frac{\pi}{6} + x\right)$$

$$\Leftrightarrow \begin{cases} 3x = \frac{\pi}{6} + x + 2k\pi & (\times) \\ 3x = \frac{5\pi}{6} - x + 2k\pi & (\pi - x) \end{cases}$$

$$\Leftrightarrow \begin{cases} 2x = \frac{\pi}{6} + 2k\pi \\ 4x = \frac{5\pi}{6} + 2k\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{\pi}{12} + k\pi \\ x = \frac{5\pi}{24} + \frac{k\pi}{2} \end{cases}$$

$S_{\sin}]-\pi, \pi]$:

$$S: \left\{ -\frac{11\pi}{12}, -\frac{19\pi}{24}, -\frac{7\pi}{24}, \frac{\pi}{12}, \frac{5\pi}{24}, \frac{13\pi}{24} \right\}$$

$$6. \sin 4x + \sin x = 0$$

$$\Leftrightarrow \sin 4x = -\sin x \Leftrightarrow \sin 4x = \sin(-x)$$

$$\Leftrightarrow \begin{cases} 4x = -x + 2k\pi \\ 4x = \pi + x + 2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} 5x = 2k\pi \\ 3x = \pi + 2k\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{2k\pi}{5} \\ x = \frac{\pi}{3} + \frac{2k\pi}{3} \end{cases}$$

$S_{\sin}]-\pi, \pi]$:

$$S: \left\{ -\frac{4\pi}{5}, -\frac{2\pi}{5}, -\frac{\pi}{3}, 0, \frac{\pi}{3}, \frac{2\pi}{5}, \frac{4\pi}{5}, \pi \right\}$$

$$7. \sin\left(3x + \frac{\pi}{6}\right) - \cos\left(x - \frac{\pi}{3}\right) = 0 \Leftrightarrow \sin\left(3x + \frac{\pi}{6}\right) = \cos\left(x - \frac{\pi}{3}\right)$$

$$\Leftrightarrow \sin\left(3x + \frac{\pi}{6}\right) = \sin\left[\frac{\pi}{2} - \left(x - \frac{\pi}{3}\right)\right]$$

$$\Leftrightarrow \sin\left(3x + \frac{\pi}{6}\right) = \sin\left(\frac{5\pi}{6} + x\right)$$

$$\Leftrightarrow \begin{cases} 3x + \frac{\pi}{6} = \frac{5\pi}{6} + x + 2k\pi \\ 3x + \frac{\pi}{6} = \frac{\pi}{6} - x + 2k\pi \end{cases} \quad \begin{matrix} \alpha \\ (\pi - \alpha) \end{matrix}$$

$$\Leftrightarrow \begin{cases} 2x = \frac{2\pi}{3} + 2k\pi \\ 4x = 2k\pi \end{cases} \Leftrightarrow \begin{cases} x = \frac{\pi}{3} + k\pi \\ x = \frac{k\pi}{2} \end{cases}$$

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$$8. 2\cos^2 x - 1 = 0 \Leftrightarrow \cos x = \pm \sqrt{\frac{1}{2}} \Leftrightarrow \cos x = \pm \frac{\sqrt{2}}{2}$$

$$\Leftrightarrow \begin{cases} x = \frac{\pi}{4} + 2k\pi \\ x = -\frac{\pi}{4} + 2k\pi \end{cases} \quad \left| \quad \begin{cases} x = -\frac{\pi}{4} + 2k\pi \\ x = \frac{3\pi}{4} + 2k\pi \end{cases}$$

$$S_{\text{un }]-\pi, \pi]: S: \left\{ -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4} \right\}$$

$$9. \sin^2 x = \cos^2 x$$

$$\Leftrightarrow \sin x = \pm \cos x$$

$$\sin x = \cos x$$

$$\Leftrightarrow \sin x = \sin\left(\frac{\pi}{2} - x\right)$$

$$\Leftrightarrow \begin{cases} x = \frac{\pi}{2} - x + 2k\pi & (u) \\ x = \frac{\pi}{2} + x + 2k\pi & (u-x) \end{cases}$$

$$\Leftrightarrow \begin{cases} 2x = \frac{\pi}{2} + 2k\pi \\ 0 = \frac{\pi}{2} + 2k\pi \Rightarrow \text{imp} \end{cases}$$

$$\Leftrightarrow x = \frac{\pi}{4} + k\pi$$

$$\sin x = -\cos x$$

$$\Leftrightarrow \sin x = \cos(\pi - x)$$

$$\Leftrightarrow \sin x = \sin\left[\frac{\pi}{2} - (\pi - x)\right]$$

$$\Leftrightarrow \sin x = \sin\left(x - \frac{\pi}{2}\right)$$

$$\Leftrightarrow \begin{cases} x = x - \frac{\pi}{2} + 2k\pi \\ x = \frac{3\pi}{2} - x + 2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} 0 = -\frac{\pi}{2} + 2k\pi \Rightarrow \text{imp} \\ 2x = \frac{3\pi}{2} + 2k\pi \end{cases}$$

$$\Leftrightarrow x = \frac{3\pi}{4} + k\pi$$

$$\text{Sow }]-\pi, \pi]: S: \left\{ -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4} \right\}$$

$$10 \quad 3 \tan \theta = 2 \cos \theta$$

$$\underline{\text{S.E.}} \quad \tan \theta \neq 0 \Rightarrow \theta \neq \frac{\pi}{2} + k\pi$$

$$3 \frac{\sin \theta}{\cos \theta} = 2 \cos \theta$$

$$\Rightarrow 3 \sin \theta - 2 \cos^2 \theta = 0$$

$$\Rightarrow 3 \sin \theta - 2(1 - \sin^2 \theta) = 0$$

$$\Rightarrow 2 \sin^2 \theta + 3 \sin \theta - 2 = 0$$

$$\Delta = 9 + 16 = 25 \Rightarrow \sin \theta_{1,2} = \frac{-3 \pm 5}{4} \begin{cases} \frac{1}{2} \\ -2 \text{ Ar.} \end{cases}$$

$$\sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6} + 2k\pi$$

$$\theta = \frac{5\pi}{6} + 2k\pi$$

$$\text{Sol }]-\pi, \pi]: S: \left\{ \frac{\pi}{6}, \frac{5\pi}{6} \right\}$$

$$\overset{x}{M} \sin \theta + \overset{x}{\sin 2\theta} + \overset{x}{\sin 3\theta} + \overset{x}{\sin 4\theta} = 0$$

$$\Leftrightarrow \left(2 \sin \frac{5\theta}{2} \cos \frac{3\theta}{2} \right) + \left(2 \sin \frac{5\theta}{2} \cos \frac{\theta}{2} \right) = 0$$

$$\Leftrightarrow \cancel{2} \sin \frac{5\theta}{2} \left(\cos \frac{3\theta}{2} + \cos \frac{\theta}{2} \right) = 0$$

$$\Leftrightarrow \sin \frac{5\theta}{2} \cancel{2} \cos \theta \cos \frac{\theta}{2} = 0$$

$$\Leftrightarrow \begin{cases} \sin \frac{5\theta}{2} = 0 \Leftrightarrow \frac{5\theta}{2} = k\pi \Leftrightarrow \theta = \frac{2k\pi}{5} \\ \cos \theta = 0 \Leftrightarrow \theta = \frac{\pi}{2} + k\pi \\ \cos \frac{\theta}{2} = 0 \Leftrightarrow \frac{\theta}{2} = \frac{\pi}{2} + k\pi \Leftrightarrow \theta = \pi + 2k\pi \end{cases}$$

$$\text{Sur }]-\pi, \pi]: S = \left\{ -\frac{\pi}{2}, \frac{\pi}{2}, \pi \right\}$$

$$CE: \theta \neq \frac{\pi}{2} + k\pi, \theta \neq \frac{\pi}{8} + k\frac{\pi}{4}$$

$$12 \tan 4\theta = 4 \tan \theta \quad (1)$$

$$\begin{aligned} \tan 4\theta &= \frac{2 \tan 2\theta}{1 - \tan^2 2\theta} = \frac{2 \frac{2 \tan \theta}{1 - \tan^2 \theta}}{1 - \left(\frac{2 \tan \theta}{1 - \tan^2 \theta} \right)^2} \\ &= \frac{\frac{4 \tan \theta}{1 - \tan^2 \theta}}{\frac{1 - 2 \tan^2 \theta + \tan^4 \theta - 4 \tan^2 \theta}{(1 - \tan^2 \theta)^2}} \\ &= \frac{4 \tan \theta (1 - \tan^2 \theta)^2}{(1 - \tan^2 \theta) (\tan^4 \theta - 6 \tan^2 \theta + 1)} \end{aligned}$$

$$\begin{aligned} (1) \Leftrightarrow \frac{4 \tan \theta (1 - \tan^2 \theta)}{\tan^4 \theta - 6 \tan^2 \theta + 1} - 4 \tan \theta &= 0 \\ \Leftrightarrow \tan \theta \left[\frac{1 - \tan^2 \theta}{1} - 1 \right] &= 0 \end{aligned}$$

$$\Leftrightarrow \tan \theta (1 - \tan^2 \theta - \tan^4 \theta + 6 \tan^2 \theta - 1) = 0$$

$$\Leftrightarrow \tan \theta (-\tan^4 \theta + 5 \tan^2 \theta) = 0$$

$$\Leftrightarrow \tan^3 \theta (5 - \tan^2 \theta) = 0$$

$$\Leftrightarrow \begin{cases} \tan \theta = 0 \Leftrightarrow \theta = k\pi \\ 5 - \tan^2 \theta = 0 \Leftrightarrow \tan \theta = \pm \sqrt{5} \end{cases}$$

$$\Leftrightarrow \theta = \pm 1,15 + k\pi$$

$$S_{\sin [-\pi, \pi]} : S: \{-1,99; -1,15; 1,15; 1,99\}$$

$$13 \quad \cos x = \sin \frac{\pi}{8}$$

$$\cos x = \cos \frac{3\pi}{8} \quad \left(\frac{\pi}{2} - \frac{\pi}{8} \right)$$

$$\Leftrightarrow \begin{cases} x = \frac{3\pi}{8} + 2k\pi \\ x = -\frac{3\pi}{8} + 2h\pi \end{cases}$$

$$\text{Sur }]-\pi, \pi]: S: \left\{ -\frac{3\pi}{8}, \frac{3\pi}{8} \right\}$$

$$14. \sin\left(3x + \frac{\pi}{3}\right) = \sin\left(\frac{2\pi}{3} - x\right)$$

$$\Leftrightarrow \begin{cases} 3x + \frac{\pi}{3} = \frac{2\pi}{3} - x + 2k\pi \\ 3x + \frac{\pi}{3} = \pi - \left(\frac{2\pi}{3} - x\right) + 2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} 4x = \frac{\pi}{3} + 2k\pi \\ 2x = 2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} x = \frac{\pi}{12} + \frac{k\pi}{2} \\ x = k\pi \end{cases}$$

$$S \cap]-\pi, \pi]: S: \left\{ -\frac{11\pi}{12}, -\frac{5\pi}{12}, 0, \frac{\pi}{12}, \frac{7\pi}{12}, \pi \right\}$$

$$15. \quad 2\sin^2 x + \sin x - 1 = 0$$

$$\Delta = 1 + 8 = 9$$

$$\sin x = \frac{-1 \pm 3}{4} \left\langle \begin{array}{l} \frac{1}{2} \\ -1 \end{array} \right.$$

$$\sin x = \frac{1}{2}$$

$$\Rightarrow x = \frac{\pi}{6} + 2k\pi$$

$$x = \frac{5\pi}{6} + 2k\pi$$

$$\sin x = -1$$

$$\Rightarrow x = \frac{3\pi}{2} + 2k\pi = -\frac{\pi}{2} + 2k\pi$$

$$S_{\sin}]-\pi, \pi] : S: \left\{ -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6} \right\}$$

$$16. \sin 2x = \tan x$$

$$\underline{CE}: x \neq \frac{\pi}{2} + k\pi$$

$$2 \sin x \cos x = \frac{\sin x}{\cos x}$$

$$\Leftrightarrow 2 \sin x \cos^2 x - \sin x = 0$$

$$\Leftrightarrow \sin x (2 \cos^2 x - 1) = 0$$

$$\Leftrightarrow \begin{cases} \sin x = 0 & \Leftrightarrow x = k\pi \\ 2 \cos^2 x - 1 = 0 & \Leftrightarrow \cos x = \pm \sqrt{\frac{2}{2}} \end{cases}$$

$$\Leftrightarrow \begin{cases} x = \pm \frac{\pi}{4} + 2k\pi \\ x = \pm \frac{3\pi}{4} + 2k\pi \end{cases}$$

$$S: \left\{ 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4} \right\} + 2k\pi$$

$$\text{Sen }]-\pi, \pi]: \left\{ -\frac{3\pi}{4}, -\frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi \right\}$$

$$17. \cos 2x = \sin x$$

$$\Leftrightarrow 1 - 2\sin^2 x - \sin x = 0$$

$$\Leftrightarrow -2\sin^2 x - \sin x + 1 = 0$$

$$\Delta = 1 + 8 = 9 \quad \sin x_{1,2} = \frac{1 \pm 3}{-4} \begin{cases} -1 \\ \frac{1}{2} \end{cases}$$

$$\sin x = -1 \Leftrightarrow x = \frac{3\pi}{2} + 2k\pi = -\frac{\pi}{2} + 2k\pi$$

$$\sin x = \frac{1}{2} \Leftrightarrow \begin{cases} x = \frac{\pi}{6} + 2k\pi \\ x = \frac{5\pi}{6} + 2k\pi \end{cases}$$

$$\text{Sol }]-\pi, \pi]: S: \left\{ -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6} \right\}$$

$$18. \frac{1 + \sin x}{1 - \sin x} = 3$$

$$\underline{CE} : 1 - \sin x \neq 0 \Leftrightarrow x \neq \frac{\pi}{2} + 2k\pi$$

$$\Leftrightarrow 1 + \sin x = 3 - 3 \sin x$$

$$\Leftrightarrow 4 \sin x - 2 = 0 \Leftrightarrow \sin x = \frac{1}{2}$$

$$\Leftrightarrow \begin{cases} x = \frac{\pi}{6} + 2k\pi \\ x = \frac{5\pi}{6} + 2k\pi \end{cases}$$

$$S_{\text{in }]-\pi, \pi]: S: \left\{ \frac{\pi}{6}, \frac{5\pi}{6} \right\}$$

$$19. \sin 2x \tan x - \tan x - \sin 2x + 1 = 0$$

$$\underline{CE}: x \neq \frac{\pi}{2} + k\pi$$

$$\Leftrightarrow \tan x (\sin 2x - 1) - (\sin 2x - 1) = 0$$

$$\Leftrightarrow (\sin 2x - 1)(\tan x - 1) = 0$$

$$\Leftrightarrow \begin{cases} \sin 2x = 1 & \Leftrightarrow 2x = \frac{\pi}{2} + 2k\pi \\ & \Leftrightarrow x = \frac{\pi}{4} + k\pi \\ \tan x = 1 & \Leftrightarrow x = \frac{\pi}{4} + k\pi \end{cases}$$

$$S_{\sin}]-\pi, \pi]: S. \left\{ -\frac{3\pi}{4}, \frac{\pi}{4} \right\}$$

$$20, \sin x + \cos x = \frac{\sqrt{2}}{2}$$

On utilise les formules en $\tan \frac{\pi}{2}$ démontrées
ou connues

Si on pose $t = \tan \frac{\pi}{2}$

$$\Rightarrow \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = \frac{\sqrt{2}}{2}$$

$$\Leftrightarrow 2t + 1 - t^2 - \frac{\sqrt{2}}{2} (1+t^2) = 0$$

$$\Leftrightarrow 4t + 2 - 2t^2 - \sqrt{2} - \sqrt{2}t^2 = 0$$

$$\Leftrightarrow -(2+\sqrt{2})t^2 + 4t + (2-\sqrt{2}) = 0$$

$$\Delta = 16 + 4(2+\sqrt{2})(2-\sqrt{2})$$

$$= 24$$

$$t_{1,2} = \frac{-4 \pm 2\sqrt{6}}{-2(2+\sqrt{2})} = -\sqrt{2} + 2 \pm (\sqrt{6} - \sqrt{3})$$

$$\cdot t_1 = \dots \Leftrightarrow \frac{\pi}{2} = -\frac{\pi}{24} + k\pi$$

$$\Leftrightarrow \pi = -\frac{\pi}{12} + 2k\pi$$

$$\cdot t_2 = \dots \Leftrightarrow \frac{\pi}{2} = \frac{7\pi}{24} + k\pi$$

$$\Leftrightarrow \pi = \frac{7\pi}{12} + 2k\pi$$

$$\text{Sur }]-\pi, \pi]: S: \left\{ -\frac{\pi}{12}, \frac{7\pi}{12} \right\}$$

$$21. 2 \sin z + 3 \cos z = 1$$

De même qu'à l'exercice précédent :

$$\frac{4t}{1+t^2} + 3 \frac{1-t^2}{1+t^2} = 1$$

$$\Leftrightarrow 4t + 3 - 3t^2 - 1 - t^2 = 0$$

$$\Leftrightarrow -4t^2 + 4t + 2 = 0$$

$$\Delta = 16 + 32 = 48$$

$$t_{1,2} = \frac{-4 \pm 4\sqrt{3}}{-8} = \frac{1 \pm \sqrt{3}}{2}$$

$$t_1 = \frac{1+\sqrt{3}}{2}$$

$$\Leftrightarrow \frac{x}{2} = 0,94 + k\pi$$

$$\Leftrightarrow x = 1,88 + 2k\pi$$

$$t_2 = \frac{1-\sqrt{3}}{2}$$

$$\Leftrightarrow \frac{x}{2} = -0,35 + k\pi$$

$$\Leftrightarrow x = -1,7 + 2k\pi$$

$$S_{\cap]-\pi, \pi]} : S = \{-1,7 ; 1,88\}$$