2. Calculer les dérivées des fonctions suivantes. Simplifier et factoriser le résultat le plus possible.

   (a) $[(x^2-6)(2x+3)^2]'$

$$= 2x(2x+3)^2 + 2(x^2-6)(2x+3).2$$

$$= 2(2x+3)[x(2x+3)+2(x^2-6)]$$

$$= 2(2x+3)(2x^2+3x+2x^2-12)$$

$$= 2(2x+3)(4x^2+3x-12)$$

(b) $\left[\left(4x-\frac{5}{x^2}\right)^2\left(-\frac{1}{2x^2}+5\right)^2\right]'$

$$= 2\left(4x-\frac{5}{x^2}\right)\left(4-5\,\frac{-2\cancel{x}}{x^{\cancel{4}3}}\right)\left(5-\frac{1}{2x^2}\right)^2 \dots$$

$$\dots + \left(4x-\frac{5}{x^2}\right)^2 2\left(5-\frac{1}{2x^2}\right)\left(-\frac{1}{2}\,\frac{-2\cancel{x}}{x^{\cancel{4}3}}\right)$$

$$= 2\left(4x-\frac{5}{x^2}\right)\left(5-\frac{1}{2x^2}\right)\dots$$

$$\dots\left[\left(4+\frac{10}{x^3}\right)\left(5-\frac{1}{2x^2}\right)+\left(4x-\frac{5}{x^2}\right)\frac{1}{x^3}\right]$$

$$= 2\left(4x-\frac{5}{x^2}\right)\left(5-\frac{1}{2x^2}\right)\left[20-\frac{2}{x^2}+\frac{50}{x^3}-\frac{5}{x^6}\dots\right.$$

$$\left.\dots+\frac{4}{x^2}-\frac{5}{x^6}\right]$$

$$= 2\left(4x-\frac{5}{x^2}\right)\left(5-\frac{1}{2x^2}\right)\left(-\frac{10}{x^6}+\frac{50}{x^3}+\frac{2}{x^2}+20\right)$$

$$= 4\left(4x-\frac{5}{x^2}\right)\left(5-\frac{1}{2x^2}\right)\left(-\frac{5}{x^6}+\frac{25}{x^3}+\frac{1}{x^2}+10\right)$$

(c) $\left[\dfrac{1-x^3}{(1-x^2)^2}\right]'$

$$= \frac{-3x^2(1-x^2)^{\cancel{2}} - (1-x^3)\,2\,\cancel{(1-x^2)}\,(-2x)}{(1-x^2)^{\cancel{4}\,3}}$$

$$= \frac{-3x^2(1-x^2) + 4x(1-x^3)}{D}$$

$$= \frac{-3x^2 + 3x^4 + 4x - 4x^4}{D}$$

$$= \frac{-x^4 - 3x^2 + 4x}{D}$$

$$= \frac{x(-x^3 - 3x + 4)}{(1-x^2)^3}$$

$$\left(= \frac{x(\overset{-1}{\cancel{x}} - 1)(-x^2 - x - 4)}{(1-x)^{\cancel{3}\,2}(1+x)^3} \right.$$

$$\left. = \frac{x(x^2 + x + 4)}{(1-x)^2(1+x)^3} \right)$$

(d) $\left[(x-1)^2\sqrt{x^2+2x-1}\right]'$

$$= 2(x-1)\sqrt{x^2+2x-1} + (x-1)^2 \cdot \frac{2x+2}{2\sqrt{\ }}$$

$$= \cancel{2}(x-1)\,\frac{2x^2+4x-2+x^2-1}{\cancel{2}\sqrt{\ }}$$

$$= (x-1)\,\frac{3x^2+4x-3}{\sqrt{\ }}$$

$$= \frac{(x-1)(3x^2+4x-3)}{\sqrt{x^2+2x-1}}$$

(e) $\left[\dfrac{2x^2-1}{\sqrt{1+x^2}}\right]'$

$$= \frac{4x\sqrt{1+x^2} - (2x^2-1)\dfrac{\cancel{2}x}{\cancel{2}\sqrt{\ }}}{1+x^2}$$

$$= \frac{4x(1+x^2) - (2x^2-1)x}{(1+x^2)\sqrt{1+x^2}}$$

$$= \frac{4x + 4x^3 - 2x^3 + x}{D}$$

$$= \frac{2x^3 + 5x}{D}$$

$$= x\,\frac{2x^2+5}{(x^2+1)\sqrt{x^2+1}}$$

$$= x\,\frac{2x^2+5}{\sqrt{(x^2+1)^3}}$$

(f) $\left[\dfrac{(x^2+1)(x-2)^4}{(x+7)^3}\right]'$

$$= \frac{\left[2x(x-2)^4 + (x^2+1)\,4\,(x-2)^3\right](x+7)^3 - \dots}{(x+7)^6}$$

$$\dots \frac{(x^2+1)(x-2)^4\;3\,(x+7)^2}{(x+7)^6}$$

$$= (x-2)^3\cancel{(x+7)^2}\,\frac{\left[2x(x-2)+4(x^2+1)\right](x+7) - \dots}{(x+7)^{\cancel{6}\,4}}$$

$$\dots \frac{3(x^2+1)(x-2)}{(x+7)^{\cancel{6}\,4}}$$

$$= (x-2)^3\,\frac{(\overbrace{2x^2-4x+4x^2}^{6x^2}+4)(x+7) - 3(x^3-2x^2+x-2)}{(x+7)^4}$$

$$= (x-2)^3\,\frac{6x^3-4x^2+4x+42x^2-28x+28-3x^3\dots}{(x+7)^4}$$

$$\dots \frac{+6x^2-3x+6}{(x+7)^4}$$

$$= (x-2)^3\,\frac{3x^3+44x^2-27x+34}{(x+7)^4}$$

(g) $\left[\frac{2}{x}\sqrt{4+x^2}\right]'$

$$= -\frac{2}{x^2}\cdot\sqrt{4+x^2} + \frac{2}{x}\cdot\frac{\cancel{2}x}{\cancel{2}\sqrt{}}$$

$$= \frac{-2(4+x^2) + 2x^2}{x^2\sqrt{}}$$

$$= \frac{-8}{x^2\sqrt{4+x^2}}$$

(h) $\left[\dfrac{x^3}{\sqrt{(1-x^2)^3}}\right]'$

$$= \frac{3x^2\sqrt{} + x^3\,\dfrac{3(1-x^2)^2(+\cancel{2}x)}{\cancel{2}\sqrt{}}}{(1-x^2)^3}$$

$$= \frac{3x^2(1-x^2)^{\cancel{3}} + 3x^4\cancel{(1-x^2)^2}}{(1-x^2)^{\cancel{3}}\sqrt{}}$$

$$= \frac{3x^2 - 3x^4 + 3x^4}{D}$$

$$= \frac{3x^2}{\sqrt{(1-x^2)^5}}$$

(i) $\left[\dfrac{x(3+2x^2)}{3\sqrt[4]{(1+x^2)^3}}\right]' = \left[\dfrac{3x+2x^3}{3\,(1+x^2)^{3/4}}\right]'$

$$= \frac{1}{3}\ \frac{(3+6x^2)(\quad)^{\frac{3}{4}} - (3x+2x^3)\cdot\frac{3}{2\cancel{4}}(1+x^2)^{-\frac{1}{4}}\cancel{2x}}{(1+x^2)^{3/2}}$$

$$= \frac{1}{3}\ \frac{2(3+6x^2)(1+x^2) - 3x(3x+2x^3)}{2(1+x^2)^{\frac{3}{2}}(1+x^2)^{\frac{1}{4}}}$$

$$= \frac{1}{6}\ \frac{2(6x^4+9x^2+3) - (9x^2-6x^4)}{(1+x^2)^{\frac{7}{4}}}$$

$$= \frac{1}{2\,\cancel{6}}\ \frac{\overset{2}{\cancel{6}}x^4 + \overset{3}{\cancel{9}}x^2 + \overset{2}{\cancel{6}}}{D}$$

$$= \frac{2x^4+3x^2+2}{2\sqrt[4]{(1+x^2)^7}}$$

(j) $\left[\frac{(1+3x^2)\sqrt{1-2x^2}}{2x^3}\right]'$

$$= \frac{\left(6x\sqrt{\ } + (1+3x^2)\cdot\frac{-4x}{2\sqrt{\ }}\right)\cancel{2}x^3 - \dots}{2\cancel{4}x^6}$$

$$\dots \frac{(1+3x^2)\sqrt{\ }\cdot\overset{3}{\cancel{6}}x^2}{2\cancel{4}x^6}$$

$$= \frac{\left[12x(1-2x^2) - 4x(1+3x^2)\right]x^3 - 6x^2(1+3x^2)(1-2x^2)}{2\cdot 2x^6\sqrt{\ }}$$

$$= x^2\,\frac{(\overbrace{12x - 24x^3 - 4x}^{8x} - 12x^3)x - 6(-6x^4 + x^2 + 1)}{D}$$

$$= \frac{8x^2 - 3\cancel{6}\cancel{x^4} + 3\cancel{6}\cancel{x^4} - 6x^2 - 6}{4x^4\sqrt{\ }}$$

$$= \frac{2x^2 - 6}{4x^4\sqrt{\ }}$$

$$= \frac{x^2 - 3}{2x^4\sqrt{1-2x^2}}$$