

Dossier de révision Conectif

Analyse : fct exp. et log.

1) $\lim_{n \rightarrow 0} (\sin n \cdot \ln n) = 0 \cdot \infty$ FI

2 $\lim_{n \rightarrow 0} \frac{\ln n}{\frac{1}{\sin n}} = \frac{\infty}{\infty}$ FI

(H) $\lim_{n \rightarrow 0} \frac{\frac{1}{n}}{-\frac{\cos n}{\sin^2 n}} = \lim_{n \rightarrow 0} -\frac{\sin^2 n}{n \cos n}$
 $= \lim_{n \rightarrow 0} -\frac{\sin n \cdot \sin n}{n \cos n} = \lim_{n \rightarrow 0} -\frac{\sin n}{\cos n} = 0$

$\lim_{x \rightarrow \infty} \frac{\ln \frac{x+1}{x}}{\ln \frac{x-1}{x}} = \frac{0}{0}$ FI (*)

car $\lim_{x \rightarrow \infty} \ln \frac{x+1}{x} = \ln \frac{\infty}{\infty} = \ln \lim_{x \rightarrow \infty} \frac{x+1}{x}$
 $= \ln \frac{\infty}{\infty}$ FI $= \ln \lim_{x \rightarrow \infty} \frac{x}{x} = \ln 1 = 0$

(idem pour $\ln \frac{x-1}{x}$)

(*) (RH) $\lim_{x \rightarrow \infty} \frac{(\quad)'}{(\quad)'}$ (**)

$\left(\ln \frac{x+1}{x}\right)' = \frac{1}{\frac{x+1}{x}} \cdot \left(\frac{x+1}{x}\right)' = \frac{x}{x+1} \cdot \frac{x - (x+1)}{x^2}$
 $= \frac{-1}{x(x+1)}$

et $\left(\ln \frac{x-1}{x}\right)' = \frac{1}{\frac{x-1}{x}} \cdot \left(\frac{x-1}{x}\right)' = \frac{x}{x-1} \cdot \frac{x - (x-1)}{x^2}$

$$(*) = \lim_{x \rightarrow \infty} \frac{x(x-1)}{x(x+1)} = \frac{\infty}{\infty} \quad \text{FI}$$

$$(*) = \lim_{x \rightarrow \infty} -1 = -1$$

$$1) \lim_1 \left(\frac{x}{x-1} - \frac{1}{\ln x} \right) = \infty - \infty \quad \text{FI}$$

$$= \lim_1 \frac{x \ln x - x + 1}{(x-1) \ln x} = \frac{0}{0} \quad \text{FI}$$

$$(*) \lim_1 \frac{\ln x + x \cdot \frac{1}{x} - 1}{\ln x + x \cdot \frac{x-1}{x}}$$

$$= \lim_1 \frac{\ln x + 1 - 1}{x \ln x + x - 1} = \frac{0}{0} \quad \text{FI}$$

$$(*) = \lim_1 \frac{\frac{1}{x} \cdot x + \ln x}{\ln x + x \cdot \frac{1}{x} + 1} = \lim_1 \frac{1 + \ln x}{2 + \ln x}$$

$$= \frac{1}{2}$$

$$2) 2^{\frac{1+3n}{2}} - 2^{\frac{1-3n}{2}} = 3$$

$$\Leftrightarrow 2 \cdot 2^{\frac{3n}{2}} - 2 \cdot 2^{-\frac{3n}{2}} = 3$$

$$\Leftrightarrow 2 \cdot 2^{\frac{3n}{2}} - \frac{2}{2^{\frac{3n}{2}}} = 3$$

$$\Leftrightarrow 2 \cdot 2^{\frac{6n}{2}} - 2 = 3 \cdot 2^{\frac{3n}{2}}$$

$$\Leftrightarrow 2 \cdot 2^{3n} - 3 \cdot 2^{\frac{3n}{2}} - 2 = 0$$

$$\Delta = 9 + 16 = 25 \quad 2^{\frac{3n}{2}} = \frac{3 \pm 5}{4} \begin{cases} 2 \\ -\frac{1}{2} \end{cases} \quad (\text{AR})$$

$$2^{3n} = 2 \quad \Leftrightarrow 3n = 1 \quad \Leftrightarrow n = \frac{1}{3}$$

$$S: \left\{ \frac{1}{3} \right\}$$

$$e^{2n-1} - \sqrt{e^{2n+1}} - 2e^3 = 0 \quad (2)$$

$$\Rightarrow e^{2n-1} - e^{n+1} - 2e^3 = 0$$

$$\Leftrightarrow \frac{e^{2n}}{e} - e e^2 - 2e^3 = 0$$

$$\Delta = e^2 + 8 \frac{e^3}{e} = 9e^2$$

$$e^{n_{1,2}} = \frac{e \pm 3e}{\frac{2}{e}} = \begin{cases} 2e^2 \\ -e^2 \end{cases} \quad \text{AR}$$

$$e^n = 2e^2 \Leftrightarrow e^n = e^{\ln 2e^2}$$

$$\Leftrightarrow n = \ln 2e^2 \Leftrightarrow n = \ln 2 + \ln e^2$$

$$\Leftrightarrow n = 2 + \ln 2$$

$$) \log^4 n - 34 \log^2 n + 225 = 0$$

$$\text{EE: } n > 0$$

$$\Delta_n = 256 \rightarrow \log^2 n = \frac{34 \pm 16}{2} \begin{cases} 25 \Rightarrow \log n = \pm 5 \\ 9 \Rightarrow \log n = \pm 3 \end{cases}$$

$$S: \{10^{-5}, 10^{-3}, 10^3, 10^5\}$$

$$) \log_2(4n^2 + n - 3) + 4 = 3 \log_2 2n - \log_2 \frac{n}{8}$$

$$\text{EE: } 4n^2 + n - 3 > 0$$

$$\Delta = 1 + 48 = 49$$

$$x_{1,2} = \frac{-1 \pm 7}{8}$$

$$\left. \begin{array}{c|ccc} n & -1 & 3/4 & \\ \hline \text{EE}_n & + & 0 & - & 0 & + \end{array} \right\} n > \frac{3}{4}$$

$$2n > 0 \Leftrightarrow n > 0$$

$$\frac{n}{8} > 0 \Leftrightarrow n > 0$$

$$\log_2(4n^2 + n - 3) + \log_2 2^4 = \log_2 (2n)^3 - \log_2 \frac{n}{8}$$

$$\Leftrightarrow \cancel{\log_2(4n^2 + n - 3)} \cdot 16 = \cancel{\log_2} \frac{8n^3}{\frac{n}{8}}$$

$$\Leftrightarrow \cancel{64n^2} + 16n - 48 = \cancel{64n^2} \Leftrightarrow n = 3 \quad S: \{3\}$$

$$\begin{cases} x+y=7 \\ \ln x + \ln y = \ln 12 \end{cases} \quad \text{c.f.: } x, y > 0 \quad (3)$$

$$\Leftrightarrow \begin{cases} y = 7-x \\ \ln x + \ln(7-x) = \ln 12 \end{cases}$$

$$\Leftrightarrow \begin{cases} (1) \\ \ln x(7-x) = \ln 12 \end{cases} \quad (*)$$

$$(*) \quad -x^2 + 7x - 12 = 0$$

$$\Delta = 49 - 48 = 1 \Rightarrow x_{1,2} = \frac{-7 \pm 1}{-2} \begin{cases} 3 \rightarrow y=4 \\ 4 \rightarrow y=3 \end{cases}$$

$$S: \{(3,4), (4,3)\}$$

$$3) \quad a) \left(\frac{x \ln x}{1 - \ln^2 x} \right)' = \frac{(\ln x + x \cdot \frac{1}{x})(1 - \ln^2 x) + x \ln x (-2 \ln x \cdot \frac{1}{x})}{(1 - \ln^2 x)^2}$$

$$= \frac{(\ln x + 1)(1 - \ln^2 x) + 2 \ln^2 x}{(1 - \ln^2 x)^2}$$

$$\stackrel{D}{=} \frac{-\ln^3 x + \ln^2 x + \ln x + 1}{(1 - \ln^2 x)^2}$$

$$b) \left(\ln \sqrt{1 - 2 \sin^2 2x} \right)' = \frac{1}{\sqrt{1 - 2 \sin^2 2x}} \cdot (\sqrt{\quad})'$$

$$= \frac{1}{\sqrt{\quad}} \cdot \frac{(1 - 2 \sin^2 2x)'}{2 \sqrt{\quad}}$$

$$= \frac{1}{2 \sqrt{1 - 2 \sin^2 2x}} \cdot (-4 \sin 2x \cos 2x \cdot 2)$$

$$= \frac{-2 \sin 4x}{1 - 2 \sin^2 2x}$$

$$c) [\ln(1+x^2) + 2x^2 \operatorname{arcsin}(\ln x)]'$$

$$= \frac{2x}{x^2+1} + 4x \operatorname{arcsin}(\ln x) + 2x \cdot \frac{1}{\sqrt{1-\ln^2 x}}$$

$$= \frac{2x}{x^2+1} + 4x \operatorname{arcsin}(\ln x) + \frac{2x}{\sqrt{1-\ln^2 x}}$$

$$d) (e^{\sqrt{x} - \ln \sqrt{x}})' = e^{\sqrt{x} - \ln \sqrt{x}} \left(\frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x}} \right)$$

$$= e^{\sqrt{x} - \ln \sqrt{x}} \cdot \left(\frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x}} \right)$$

$$= e^{\sqrt{x} - \ln \sqrt{x}} \cdot \frac{\sqrt{x} - 1}{2\sqrt{x}}$$

$$4) a) \text{CE: } x + \sqrt{x^2+1} > 0$$

$$\Leftrightarrow x > -\sqrt{x^2+1}$$

$$\text{Si } x > 0 \Rightarrow \text{tp vrai} \Rightarrow S: \mathbb{R}_0^+$$

$$\text{Si } x < 0 \Rightarrow x^2 < x^2+1 \Leftrightarrow 0 < 1$$

$$\Rightarrow \text{tp vrai} \Rightarrow S: \mathbb{R}_0^-$$

$$\text{Si } x = 0 \Rightarrow 0$$

$$\Rightarrow \text{dom } f: \mathbb{R}$$

$$b) \text{CE: } x > 0$$

$$\ln^2 x - \ln x \geq 0 \Leftrightarrow \ln x (\ln x - 1) \geq 0$$

		0	1	$x=1$ e	
x	/				
ln x	/	-	0	+	
ln x - 1	/	-	-	0	+
CE	/	+	0	-	+

$$\Rightarrow \text{dom } f:]0, 1] \cup [e, +\infty$$

$$c) f(x) = \ln \frac{1+\sin x}{1-\sin x}$$

$$T = 2\pi$$

(4)

$$\underline{CE} : \frac{1+\sin x}{1-\sin x} > 0 \quad (1)$$

$$1-\sin x \neq 0 \Leftrightarrow \sin x \neq 1 \Leftrightarrow x \neq \frac{\pi}{2} + 2k\pi$$

(1)	x	$-\pi$	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	π
	$1+\sin x$	+	0	+	+
	$1-\sin x$	+	+	0	+
	CE	+	0	+	+

$$\text{dom}_f : \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi \mid k \in \mathbb{Z} \right\}$$

5) Voir feuille annexes

$$3. f(x) = 2x + \frac{e^x}{e^x - 2}$$

- $\text{dom } f : \mathbb{C} \setminus \{e^x - 2 \neq 0\} \Leftrightarrow x \neq \ln 2 \Rightarrow \text{dom } f : \mathbb{R} \setminus \{\ln 2\}$
- zéros : —
- $\text{NO}_y : f(0) = -1$
- signe : voir page suivante (difficile)
- AV : $\lim_{x \rightarrow \ln 2} f(x) = 2 \ln 2 + \frac{2}{0} = \pm \infty \Rightarrow \text{AV} \leq x = \ln 2$

$$\text{AH} : \lim_{x \rightarrow +\infty} f(x) = \infty + \frac{\infty}{\infty} \quad \textcircled{\text{FH}} \\ \hookrightarrow \textcircled{\text{FI}} = \lim_{x \rightarrow +\infty} \frac{e^x}{e^x} = 1 \quad (*)$$

$$\Rightarrow \lim_{x \rightarrow +\infty} f(x) = +\infty \Rightarrow \text{AH}_d \quad \checkmark$$

$$* \lim_{x \rightarrow -\infty} f(x) = -\infty + \frac{0}{-2} = -\infty \Rightarrow \text{AH}_g \quad \checkmark$$

$$\text{AO} : m = \lim_{x \rightarrow +\infty} \frac{f(x)}{x} = \lim_{x \rightarrow +\infty} \left(2 + \frac{e^x}{x(e^x - 2)} \right)$$

$$= 2 + \frac{\infty}{\infty} \quad \textcircled{\text{FI}}$$

$$\hookrightarrow \textcircled{\text{FI}} \quad \textcircled{\text{FH}} \quad \lim_{x \rightarrow +\infty} \frac{e^x}{x e^x + e^x - 2} \\ = \lim_{x \rightarrow +\infty} \frac{e^x}{e^x (x + 1 - 2e^{-x})} = 0$$

$$p = \lim_{x \rightarrow +\infty} \left(2x + \frac{e^x}{e^x - 2} - 2x \right) = \lim_{x \rightarrow +\infty} \frac{e^x}{e^x - 2} = 1 \quad \neq (*)$$

$$\Rightarrow \text{AO}_d : y = 2x + 1$$

$$*m = \lim_{n \rightarrow -\infty} \frac{f(n)}{n} = \lim_{n \rightarrow -\infty} \left(2 + \frac{e^n}{n(e^n - 2)} \right)$$

$$= 2 + \frac{0}{-\infty(-2)} = 2$$

$$p = \lim_{n \rightarrow -\infty} \left(\frac{e^n}{e^n - 2} \right) = \frac{0}{-2} = 0 \Rightarrow A_{0g} = y = 2n$$

$$f'(n) = 2 + \frac{e^n(e^n - 2) - e^{2n}}{(e^n - 2)^2}$$

$$= \frac{2(e^{2n} - 4e^n + 4) + e^{2n} - 2e^n - e^{2n}}{(e^n - 2)^2}$$

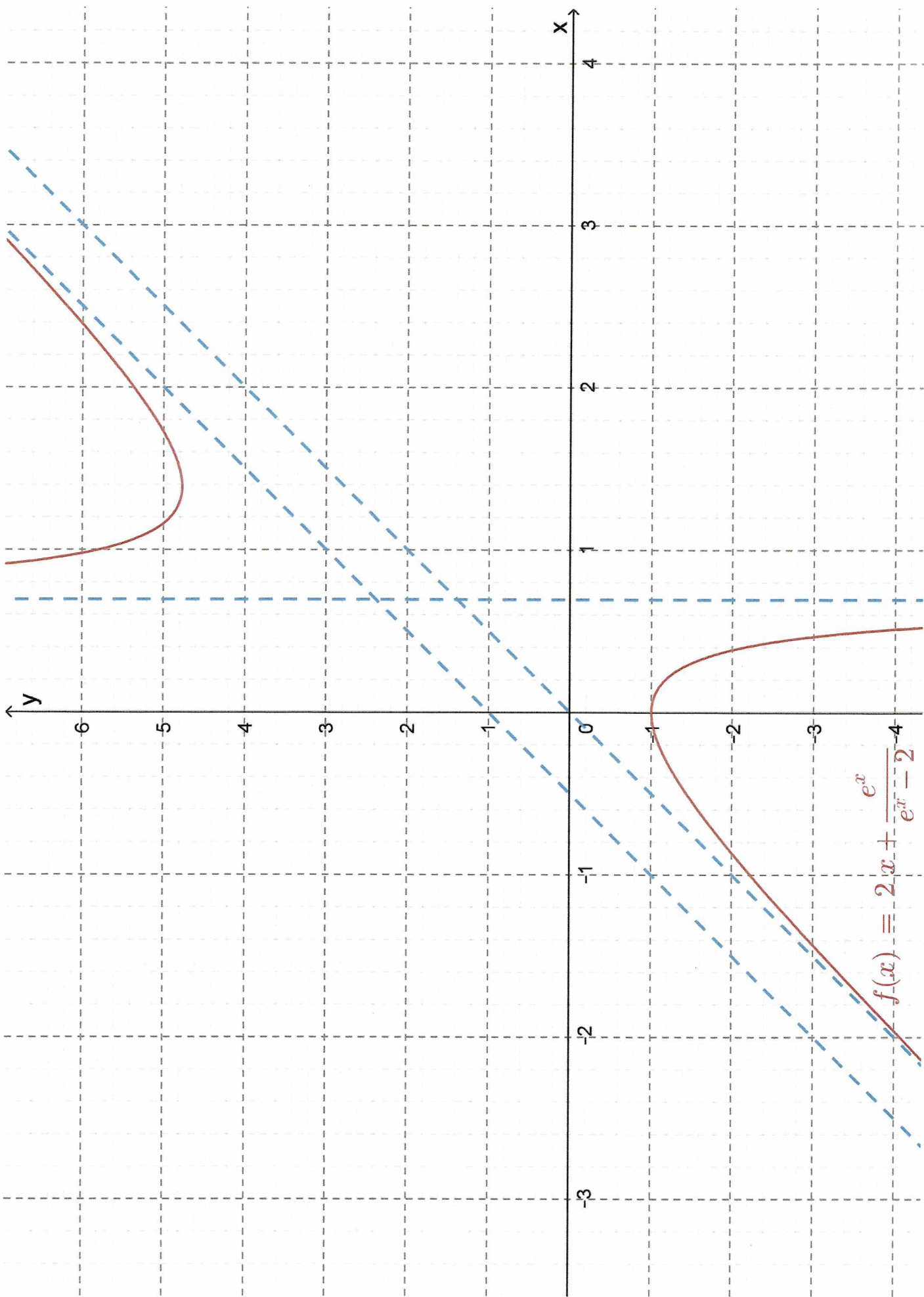
$$= \frac{2e^{2n} - 8e^n + 8 - 2e^n}{(e^n - 2)^2}$$

$$= 2 \frac{e^{2n} - 5e^n + 4}{(e^n - 2)^2}$$

Zero N: $e^{2n} - 5e^n + 4 = 0$

$$\Delta = 25 - 16 = 9 \quad e^{n_{1/2}} = \frac{5 \pm 3}{2} < \frac{4}{1}$$

$\begin{cases} e^n = 4 \\ e^n = 1 \end{cases}$	$\Leftrightarrow$	$\begin{cases} n = \ln 4 \\ n = 0 \end{cases}$	$\Leftrightarrow$	$\begin{cases} n = 2 \ln 2 \\ n = 0 \end{cases}$				
n		0	$\ln 2$	$2 \ln 2$				
N		+	0	-	-	0	+	
D		+		+	0	+	+	
$f'(x)$		+	0	-	AV	-	0	+
$f(x)$		$\nearrow$	$\cap$	$\searrow$		$\searrow$	m	$\nearrow$
			$(0, -1)$				$(2 \ln 2, 4 \ln 2 + 2)$	



4. $f(x) = x + 1 - e^x$ (zéro = évident)

• dom_f : $\mathbb{R}$

• zéro : $x = 0$

• Noy : $f(0) = 0$

• AV —

AM $\lim_{+\infty} f(x) = \infty - \infty = -\infty$ car e^x tend + vite vers l'infini que x

$$\lim_{-\infty} f(x) = -\infty$$

→ AM

AO inclim $\lim_{+\infty} \frac{f(x)}{x} = \lim_{+\infty} \left[1 + \frac{1}{x} - \frac{e^x}{x} \right]$
 $\hookrightarrow \frac{\infty}{\infty}$ (FI)

(LH) $= \lim_{+\infty} \frac{e^x}{1} = +\infty$

→ m $\neq$ —, AO_d —

$$m = \lim_{-\infty} \frac{f(x)}{x} = \lim_{-\infty} \left[1 + \frac{1}{x} - \frac{e^x}{x} \right] = 1$$

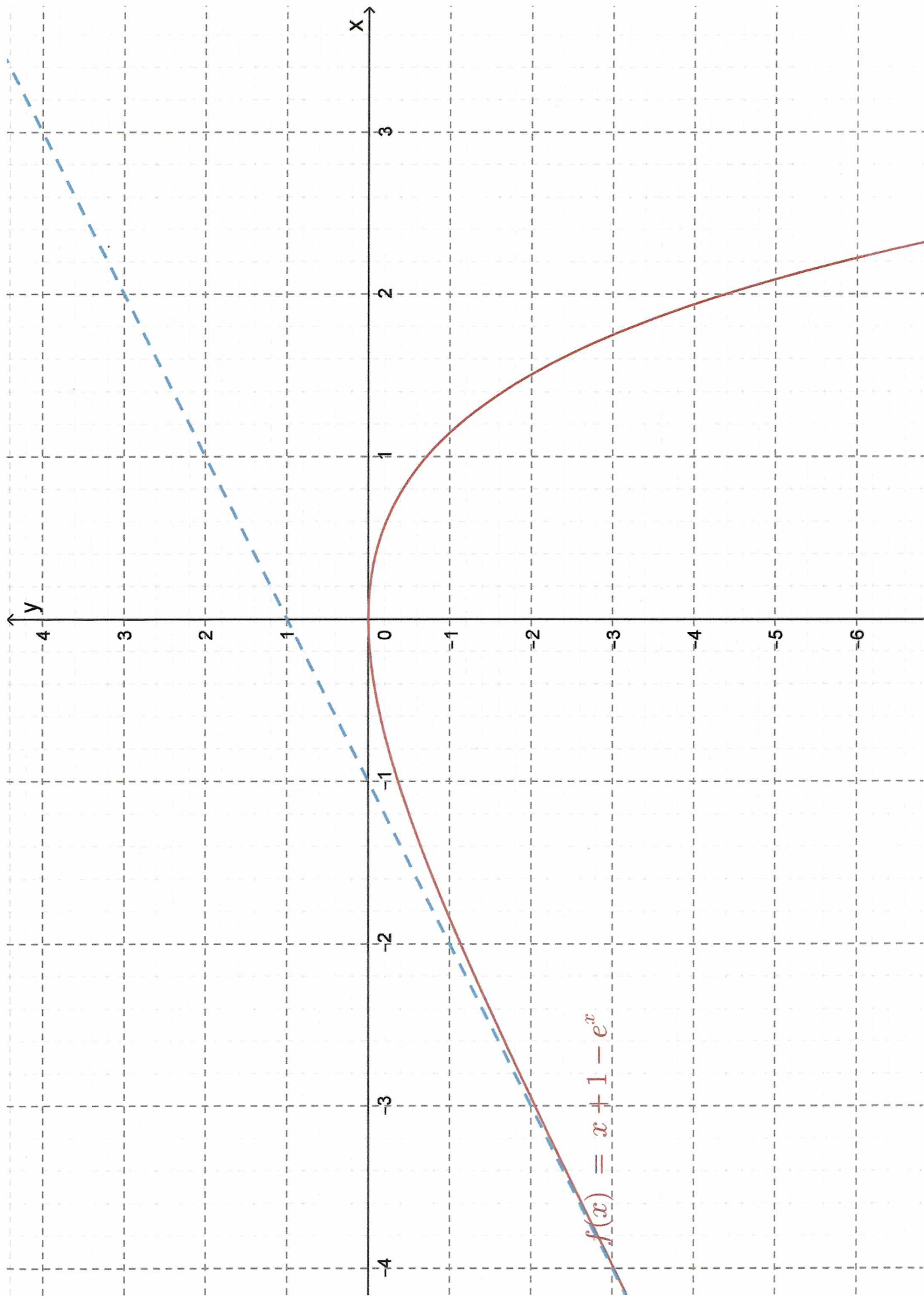
$$p = \lim_{-\infty} [f(x) - mx] = \lim_{-\infty} (1 - e^x) = 1$$

→ AO $\equiv y = x + 1$

• $f'(x) = 1 - e^x$

	x	0	
$f'(x)$	+	0	-
$f(x)$	↗	↗	↘
		(0,0)	

• $f''(x) = -e^x < 0 \Rightarrow \cap$ partout.



5. $f(x) = x e^{\frac{2x}{x^2-1}}$ (uniquement les asymptotes)

• dom f : $\mathbb{R} \setminus \{-1, 1\}$

• zéros : $x=0$

• Noy : $y=0$

• Asymptotes

AV:

x	-1	0	1
$2x$	-	0	+
x^2-1	+	-	+
$\frac{2x}{x^2-1}$	$-\infty$	0	$+\infty$

$$\lim_{x \rightarrow -1^-} f(x) = -1 \cdot e^{-\infty} = 0 \rightarrow \text{pt creux}$$

$$\lim_{x \rightarrow -1^+} f(x) = -1 \cdot e^{+\infty} = -\infty \Rightarrow \text{AV}_d = x = -1$$

$$\text{Idem : } \lim_{x \rightarrow 1^-} f(x) = 0 \text{ et } \lim_{x \rightarrow 1^+} f(x) = +\infty$$

$$\underline{\text{AH}} : \lim_{\pm\infty} f(x) = \infty \cdot e^{\frac{\infty}{\infty}} \rightarrow \textcircled{\text{FI}}$$

$$\lim_{\pm\infty} \frac{2x}{x^2-1} = \frac{\infty}{\infty} \textcircled{\text{FI}} \stackrel{\textcircled{\text{RH}}}{=} \lim_{\pm\infty} \frac{2}{2x} = 0 \quad (*)$$

$$\Rightarrow \lim_{\pm\infty} f(x) = \infty \cdot e^0 = \pm\infty \quad \text{AH} \quad /$$

$$\underline{\text{AO}} : m = \lim_{\pm\infty} \frac{f(x)}{x} = \lim_{\pm\infty} e^{\frac{2x}{x^2-1}} = e^0 = 1 \quad \text{f} \quad (*)$$

$$p = \lim_{\pm\infty} [f(x) - mx]$$

$$= \lim_{\pm\infty} x e^{\frac{2x}{x^2-1}} - x = \lim_{\pm\infty} [x(e^{\frac{2x}{x^2-1}} - 1)]$$

On a un (❌) que si $x \rightarrow \pm\infty, e^x \rightarrow 1$
 $\Rightarrow p = \infty \cdot 0$ FI

On a $p = \lim_{\pm\infty} \frac{e^x - 1}{\frac{1}{x}} = \frac{0}{0}$ (FI)

(RH) $= \dots$ (à suivre...) (❌)

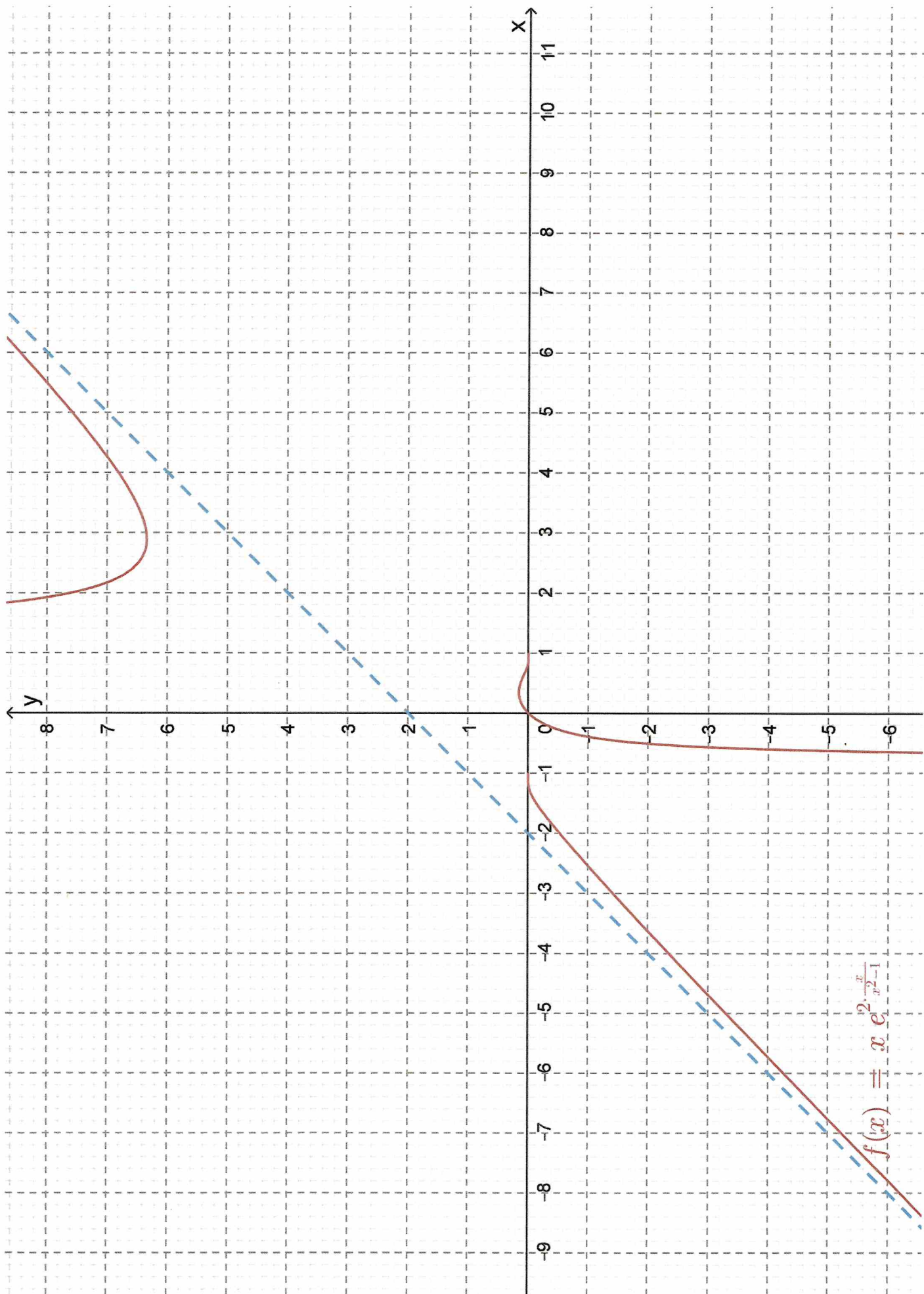
$$\begin{aligned} \left(e^{\frac{2x}{x^2-1}} \right)' &= \left(\frac{2x}{x^2-1} \right)' e^{\dots} \\ &= \frac{2(x^2-1) - 2x(2x)}{(x^2-1)^2} \cdot \frac{2x^2-2-4x^2}{1} e^{\dots} \\ &= -2 \frac{x^2+1}{(x^2-1)^2} e^{\dots} \end{aligned}$$

$$\begin{aligned} (*) &= \lim_{\pm\infty} \frac{-2 \frac{x^2+1}{(x^2-1)^2} e^{\dots}}{-\frac{1}{x^2}} = \lim_{\pm\infty} \frac{2x^2(x^2+1)}{(x^2-1)^2} \cdot e^{\dots} \\ &\quad L, \frac{\infty}{\infty} \text{ (FI)} \end{aligned}$$

$$= \lim_{\pm\infty} \frac{2x^2 x^2}{x^4} = 2$$

$$\Rightarrow p = 2 \cdot 1 = 2$$

$$\Rightarrow A0 \equiv y = x + 2$$



14. $f(x) = x e^{-\frac{x^2}{2}}$

dom f : $\mathbb{R}$

zéro : $x=0$

$\wedge \circ y : y=0$

signe

x		0	
$f(x)$	-	0	+

Av /

AM $\lim_{x \rightarrow \infty} f(x) = 0 \cdot \infty$ **(RI)** $= \lim_{x \rightarrow \infty} \frac{x}{e^{\frac{x^2}{2}}} = \frac{\infty}{\infty}$ **(FI)**

(RM) $= \lim_{x \rightarrow \infty} \frac{1}{\cancel{x} e^{\frac{x^2}{2}}} = \frac{1}{\infty} = 0 \Rightarrow AM \equiv y=0$

Ad /

$f'(x) = e^{-\frac{x^2}{2}} + x \cdot \cancel{\frac{-2x}{2}} e^{-\frac{x^2}{2}}$

$= e^{-\frac{x^2}{2}} (1 - x^2)$

x		-1		1	
$f'(x)$	-	0	+	0	-
$f(x)$	$\searrow$	$\cap$	$\nearrow$	$\cap$	$\searrow$
		$(-1, \frac{-1}{\sqrt{e}})$		$(1, \frac{1}{\sqrt{e}})$	

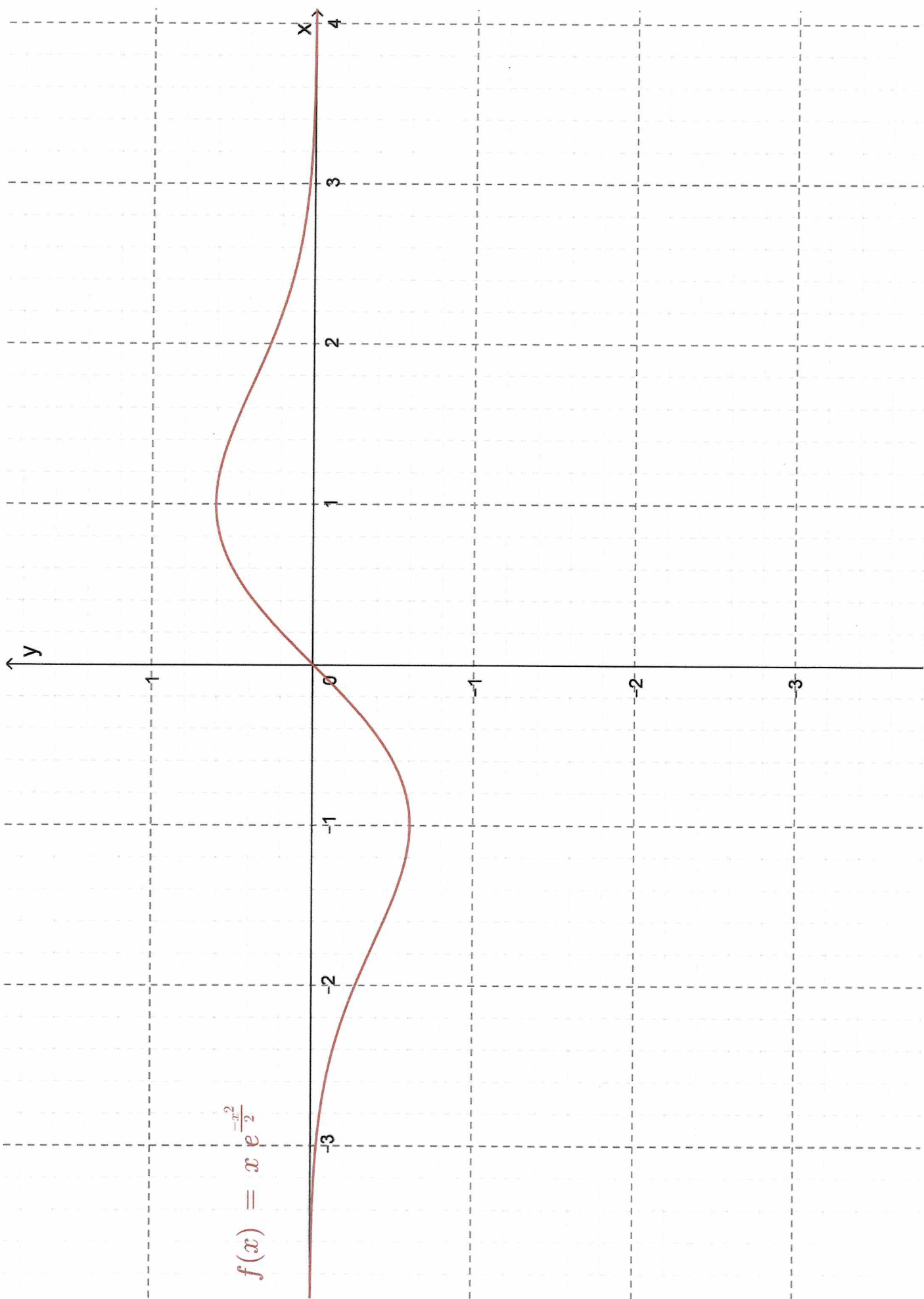
$f''(x) = -\cancel{\frac{x}{2}} e^{-\frac{x^2}{2}} (1 - x^2) - 2x e^{-\frac{x^2}{2}}$

$= -x e^{-\frac{x^2}{2}} (1 - x^2 + 2)$

$= x e^{-\frac{x^2}{2}} (x^2 - 3)$

x	$-\sqrt{3}$	0	$\sqrt{3}$				
x	-	-	0	+	+		
x^2-3	+	0	-	-	0	+	
$f'(x)$	-	0	+	0	-	0	+
$f(x)$	$\cap \text{PI} \cup \text{PI} \cap \text{PI} \cup$ $(-\sqrt{3}, -\frac{\sqrt{3}}{\sqrt{e^3}}) (0,0) (-\sqrt{3}, \frac{\sqrt{3}}{\sqrt{e^3}})$						

$$f(x) = x e^{\frac{-x^2}{2}}$$



15.16. $f(x) = \frac{\ln x}{\ln^2 x + \ln x + 1}$

dom f : $\mathbb{R}_0^+$ ($D \neq \emptyset$ typ)

zéro : $x=1$

noy /

signe

x	0	1
$f(x)$	-	0

AV : $\lim_{0^+} f(x) = \frac{\infty}{\infty}$ (FI) $\stackrel{(RM)}{=} \lim_{0^+} \frac{\frac{1}{x}}{2 \ln x \cdot \frac{1}{x} + \frac{1}{x}}$

$= \lim_{0^+} \frac{1}{2 \ln x + 1} = 0 \Rightarrow$ pt creux

AM $\lim_{+\infty} f(x) = \frac{\infty}{\infty}$ (FI) $= \dots$ $\neq$ AV $= 0$

$\rightarrow$ AM $= y = 0$

AO /

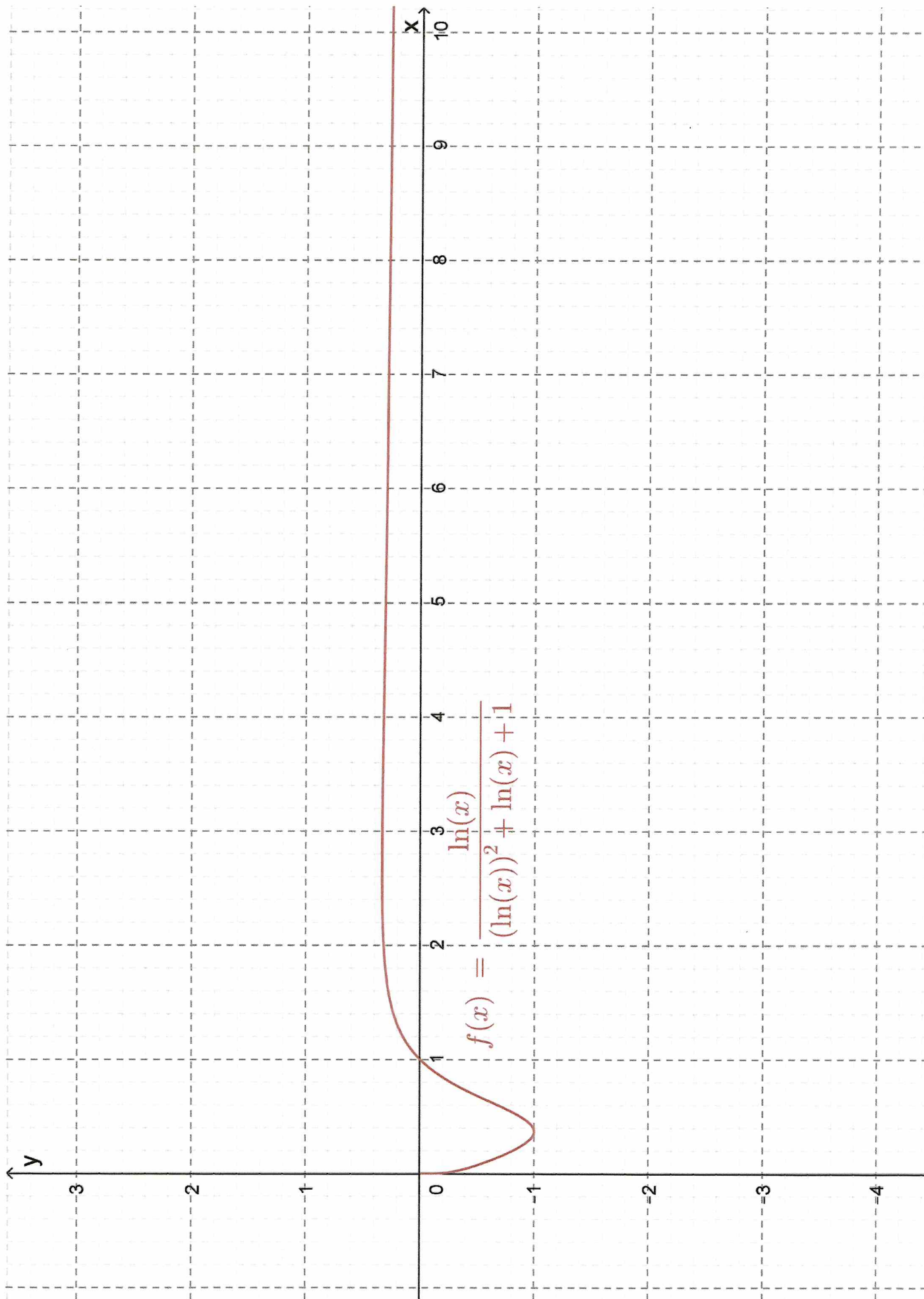
$f'(x) = \frac{\frac{1}{x} (\ln^2 x + \ln x + 1) - \ln x (2 \ln x \cdot \frac{1}{x} + \frac{1}{x})}{(\ln^2 x + \ln x + 1)^2}$

$= \frac{1}{x} \frac{\ln^2 x + \ln x + 1 - 2 \ln^2 x - \ln x}{D}$

$= \frac{-\ln^2 x + 1}{x (\ln^2 x + \ln x + 1)^2}$

$= \frac{(1 - \ln x)(1 + \ln x)}{x (\ln^2 x + \ln x + 1)^2}$

x	0	$\frac{1}{e}$	1	e
$f'(x)$	-	0	+	0
$f(x)$	$\downarrow$	m	$\uparrow$	M
		$(\frac{1}{e}, -1)$		$(e, \frac{1}{3})$



22. $f(x) = \ln \frac{x-1}{x+1}$

dom $f : \frac{x-1}{x+1} > 0$

x	-1	1
$x-1$	-	- 0 +
$x+1$	- 0 +	+ +
$\frac{x-1}{x+1}$	+	- 0 +

dom $f : -\infty, -1[\cup]1, +\infty$

zéro: $\ln \frac{x+1}{x-1} = 0 \Leftrightarrow \frac{x+1}{x-1} = 1 \Leftrightarrow x+1 = x-1$
imp

Boy ✓

signe: $f(x) > 0$ si $\frac{x-1}{x+1} > 1 \Leftrightarrow \frac{x-1-(x+1)}{x+1} > 0$

$\Leftrightarrow \frac{-2}{x+1} > 0$

x	-1
$\frac{-2}{x+1}$	+

$\Rightarrow f(x) > 0$ si $x < -1$ et inversement

x	-1	1
$f(x)$	+	-

AV: $\lim_{x \rightarrow -1} f(x) = \ln \frac{-2}{0} = +\infty$

$\lim_{x \rightarrow 1} f(x) = \ln \frac{0}{2} = -\infty$

AM $\lim_{x \rightarrow \pm\infty} f(x) = \ln \frac{\infty}{\infty}$

$\hookrightarrow \textcircled{\text{FI}} = \lim_{x \rightarrow \pm\infty} \frac{x-1}{x+1} = 1$

$\Rightarrow \text{AM} \equiv y = \ln 1 \Leftrightarrow y = 0$

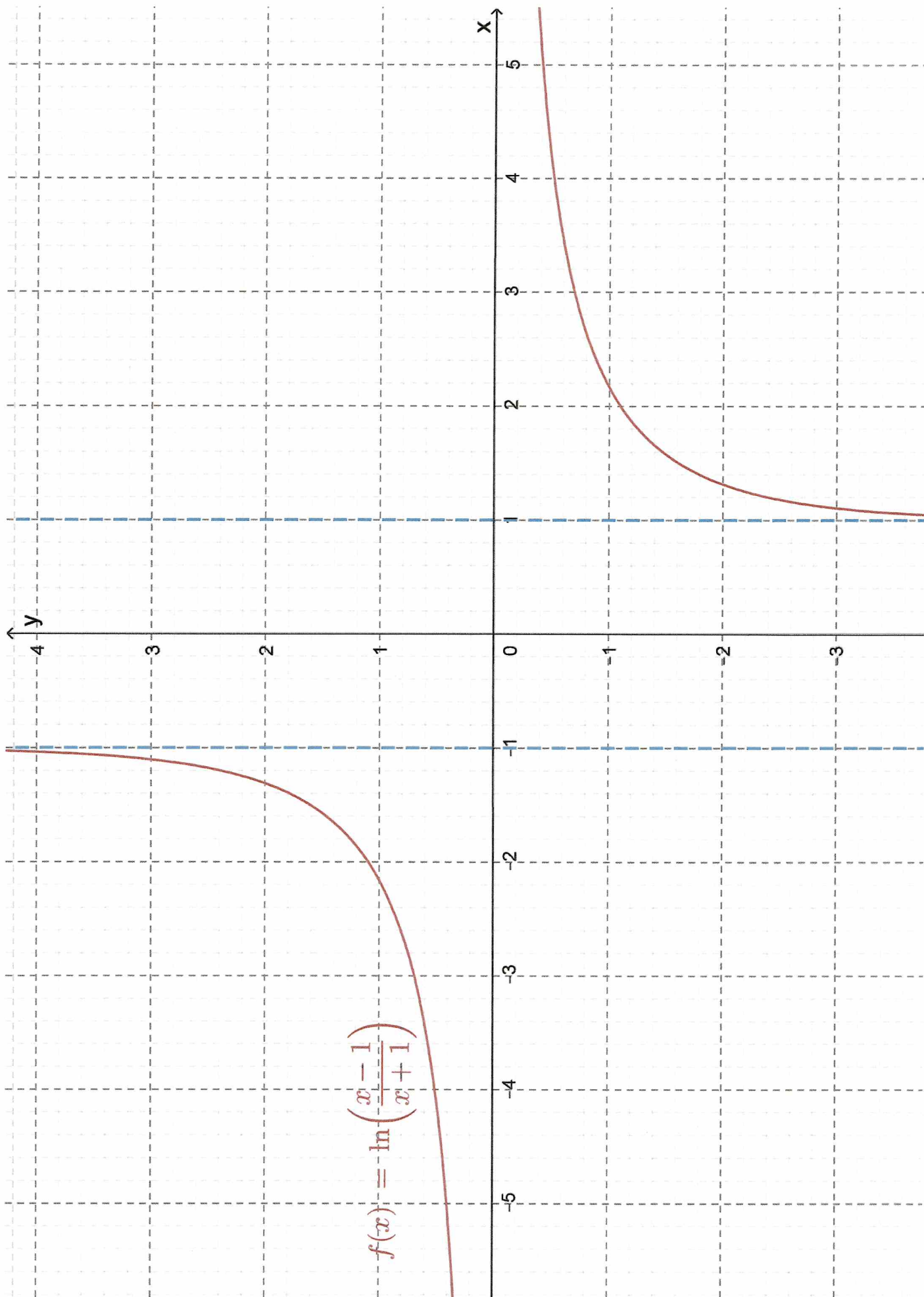
AO ✓

$$f'(x) = \frac{\left(\frac{x-1}{x+1}\right)'}{\left(\frac{x-1}{x+1}\right)} = \frac{\cancel{x+1}}{x-1} \cdot \frac{x+1 - (x-1)}{(x+1)^{\cancel{2}}} = \frac{2}{x^2-1}$$

x		-1		1	
$f'(x)$	+	0		0	+
$f(x)$	$\nearrow$	AV		AV	$\nearrow$

$$f''(x) = - \frac{2 \cdot 2x}{(x^2-1)^2}$$

x		-1		0		1	
$f''(x)$	+	0		0		0	-
$f(x)$	$\cup$						$\cap$



$$32. f(x) = \frac{e^{x+1}}{e^x - 1} - x$$

$$\cdot \text{dom } f: \mathbb{R}_0$$

$$\cdot \text{zéros: } \text{---}$$

$$\cdot \text{asy: } \text{---}$$

$$\cdot \text{signe: } \text{---}$$

$$\cdot \underline{\text{Av}}: \lim_{x \rightarrow 0} f(x) = \frac{e}{0} - 0 = \pm \infty$$

$$\underline{\text{Ah}} \lim_{x \rightarrow -\infty} f(x) = \frac{0}{-1} + \infty = \infty \Rightarrow \text{Ah}_g \quad /$$

$$\lim_{x \rightarrow +\infty} f(x) = \frac{\infty}{\infty} - \infty = \textcircled{\text{FI}}$$

$$= \lim_{x \rightarrow +\infty} \frac{e^{x+1} - x e^x - x}{e^x - 1} = \frac{\infty}{\infty} \quad \textcircled{\text{FI}}$$

$$\textcircled{\text{RH}} = \lim_{x \rightarrow +\infty} \frac{e^{x+1} - e^x - x e^x - 1}{e^x}$$

$$= \lim_{x \rightarrow +\infty} \left[\frac{\cancel{e^x} e}{\cancel{e^x}} - \frac{\cancel{e^x}}{\cancel{e^x}} - \frac{x \cancel{e^x}}{\cancel{e^x}} - \frac{1}{e^x} \right]$$

$$= e - 1 - \infty - 0 = -\infty \Rightarrow \text{Ah}_d \quad /$$

$$\underline{\text{AO}}: \frac{\infty}{\infty} = \infty$$

$$m = \lim_{x \rightarrow -\infty} \frac{f(x)}{x} = \frac{0}{-1 \cdot x} - 1 = -1$$

$$p = \lim_{x \rightarrow -\infty} [f(x) - mx] = \lim_{x \rightarrow -\infty} \frac{e^{x+1}}{e^x - 1} = 0$$

$$\Rightarrow \text{AO}_g \equiv y = -x$$

on $\rightarrow \infty$

$$m = \lim_{n \rightarrow \infty} \frac{f(n)}{n} = \lim_{n \rightarrow \infty} \frac{e e^n - n(e^n - 1)}{n(e^n - 1)}$$

$$= \lim_{n \rightarrow \infty} \frac{e \cdot e^n - n e^n + n}{n e^n - n} = \frac{\infty}{\infty} \quad (\text{FI})$$

$$\stackrel{(\text{RH})}{=} \lim_{n \rightarrow \infty} \frac{e e^n - e^n - n e^n + 1}{e^n + n e^n - 1} = \frac{\infty}{\infty} \quad (\text{FI})$$

$$\stackrel{(\text{LH})}{=} \lim_{n \rightarrow \infty} \frac{\cancel{e e^n} - \cancel{e^n} - \cancel{n e^n} - \cancel{n e^n}}{\cancel{e^n} + \cancel{e^n} + \cancel{n e^n}}$$

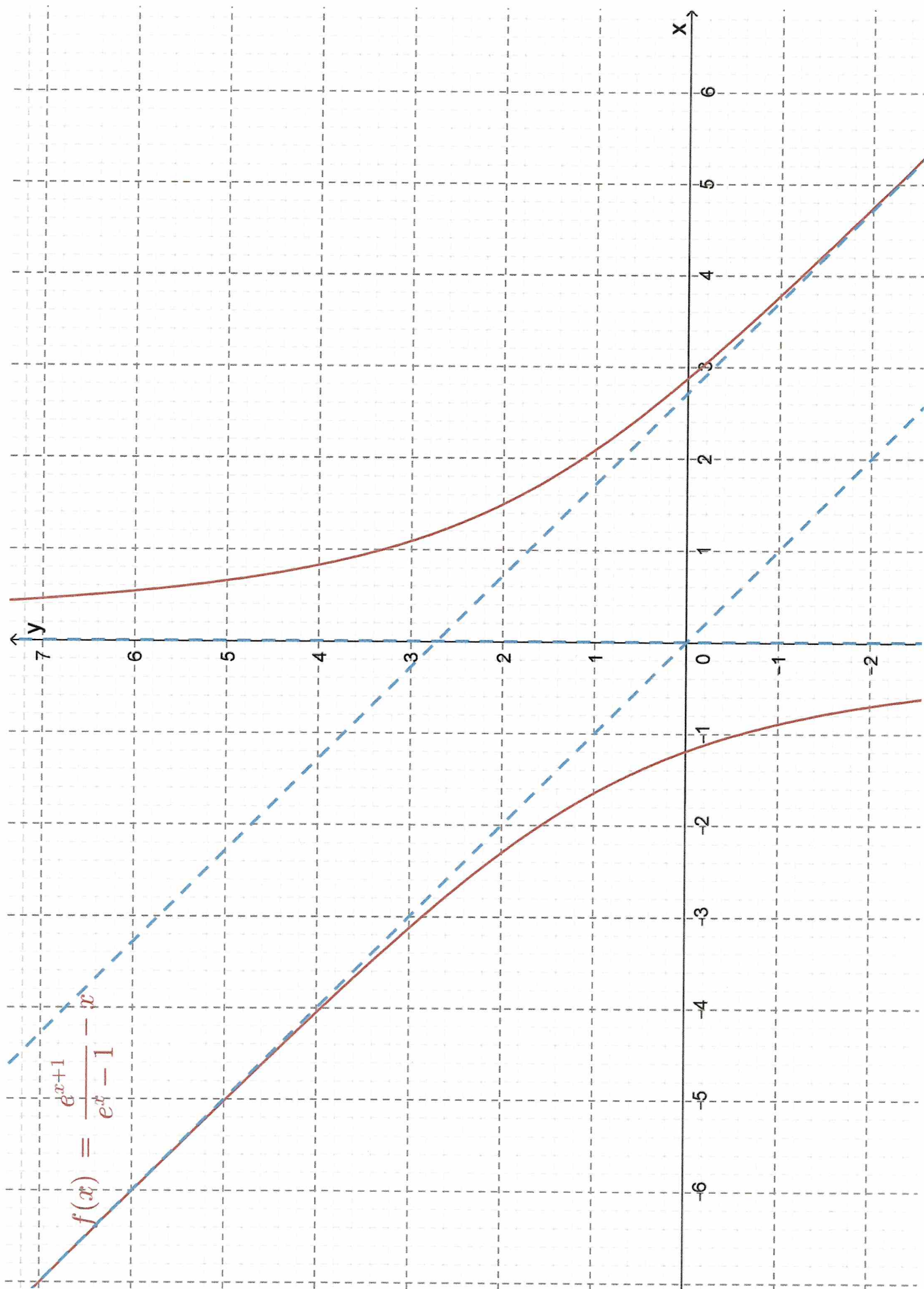
$$= \lim_{n \rightarrow \infty} \frac{e - 2 - n}{2 + n} = \frac{\infty}{\infty} \quad (\text{FI})$$

$$= \lim_{n \rightarrow \infty} \frac{-n}{n} = -1$$

$$p = \lim_{n \rightarrow \infty} \frac{e^{n+1}}{e^n - 1} = \frac{\infty}{\infty} \quad (\text{FI})$$

$$\stackrel{(\text{RH})}{=} \lim_{n \rightarrow \infty} \frac{e^{n+1}}{e^n} = e$$

$$\Rightarrow A0_d \equiv y = -n + e$$



$$f(x) = \frac{e^{x+1} - x}{e^x - 1}$$

Analyse : Primitives

on substitution $t = 2x+1$

$$\begin{aligned} 1) \int \frac{4x+5}{(2x+1)^2} dx &= \int \frac{4x}{D} dx + 5 \int \frac{1}{D} dx \\ &= 4 \int \frac{x}{(2x+1)^2} + \frac{5}{2} \int \frac{2}{(2x+1)^2} dx \\ &= 4 I_1 + \frac{5}{2} \int 2 (2x+1)^{-2} dx \\ &= 4 I_1 + \frac{5}{2} \frac{(2x+1)^{-1}}{-1} = 4 I_1 - \frac{5}{2(2x+1)} \end{aligned}$$

$$\begin{aligned} \frac{I_1}{I_1} : \frac{x}{(2x+1)^2} &= \frac{A}{2x+1} + \frac{B}{(2x+1)^2} \\ &= \frac{(2x+1)A + B}{(2x+1)^2} \end{aligned}$$

$$x = -\frac{1}{2} \Rightarrow B = -\frac{1}{2}$$

$$x = 0 \Rightarrow A + B = 0 \Leftrightarrow A = \frac{1}{2}$$

$$\begin{aligned} I_1 &= \frac{1}{2} \int \frac{2^{\frac{1}{2}}}{2x+1} dx + \frac{1}{2} \int \frac{-\frac{1}{2} 2}{(2x+1)^2} dx \\ &= \frac{1}{4} \ln |2x+1| + \frac{1}{4} \frac{1}{2x+1} \end{aligned}$$

$$\begin{aligned} \Rightarrow \int \frac{4x+5}{(2x+1)^2} dx &= \ln |2x+1| + \frac{1}{2x+1} - \frac{5}{2(2x+1)} + C \\ &= \ln |2x+1| - \frac{3}{2(2x+1)} + C \end{aligned}$$

$$2) I = \int \frac{x^2}{\sqrt{x+1}} dx \quad \text{on pose } t = x+1$$

$\Leftrightarrow \begin{cases} x = t-1 \\ dx = dt \end{cases}$

$$\begin{aligned} I &= \int \frac{(t-1)^2}{\sqrt{t}} dt = \int \frac{t^2}{\sqrt{t}} dt - 2 \int \frac{t}{\sqrt{t}} dt + \int \frac{1}{\sqrt{t}} dt \\ &= \int t^{\frac{3}{2}} dt - 2 \int t^{\frac{1}{2}} dt + \int t^{-\frac{1}{2}} dt \end{aligned}$$

$$= \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - 2 \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$

⑤

$$= \frac{2 \sqrt{t^5}}{5} - 4 \frac{\sqrt{t^3}}{3} + 2 \sqrt{t} + C$$

$$= \frac{2}{5} \sqrt{(x+1)^5} - 4 \frac{\sqrt{(x+1)^3}}{3} + 2 \sqrt{x+1} + C$$

3) I = $\int \arccos 2x \, dx$

$$\begin{cases} f = \arccos 2x \\ g' = 1 \end{cases}$$

$$\begin{cases} f' = \frac{-2}{\sqrt{1-4x^2}} \\ g = x \end{cases}$$

$$I = x \arccos 2x + \frac{2}{-2} \int \frac{-2x}{\sqrt{1-4x^2}} \, dx$$

$$= x \arccos 2x - \frac{1}{4} \frac{\sqrt{1-4x^2}}{\frac{1}{2}} + C$$

$$= x \arccos 2x - \frac{1}{2} \sqrt{1-4x^2} + C$$

4) I = $\int \frac{2x^2 + 2x + 1}{x(x^2 + 4)} \, dx$

$$\frac{2x^2 + 2x + 1}{x(x^2 + 4)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$$

$$= \frac{A(x^2 + 4) + (Bx + C)x}{x(x^2 + 4)}$$

$$x=0 \Rightarrow 1 = 4A \Rightarrow A = \frac{1}{4}$$

$$x=1 \Rightarrow 5 = 5A + B + C \Rightarrow B + C = \frac{15}{4} \quad (1)$$

$$x=-1 \Rightarrow 1 = 5A - (-B + C) \Rightarrow B - C = -\frac{1}{4} \quad (2)$$

$$(1) + (2) \Rightarrow 2B = \frac{14}{4} \Rightarrow B = \frac{7}{4}$$

$$(1) - (2) \Rightarrow 2C = \frac{16}{4} \Rightarrow C = 2$$

$$I = \frac{1}{4} \int \frac{dx}{x} + \frac{7}{4} \int \frac{x \, dx}{x^2 + 4} + 2 \int \frac{dx}{x^2 + 4}$$

$$= \frac{1}{4} \ln|x| + \frac{7}{4 \cdot 2} \int \frac{2x}{x^2+4} dx + I_3$$

$$= \frac{1}{4} \ln|x| + \frac{7}{8} \ln(x^2+4) + I_3$$

$$I_3 = 2 \int \frac{dx}{x^2+4} = 2 \int \frac{dx}{4 \left[\frac{x^2}{4} + 1 \right]} = \frac{1 \cdot 2}{2} \int \frac{\frac{1}{2} dx}{\left(\frac{x}{2}\right)^2 + 1}$$

$$= \arctan \frac{x}{2} + C$$

$$I = \frac{1}{4} \ln|x| + \frac{7}{8} \ln(x^2+4) + \arctan \frac{x}{2} + C$$

$$5) \int \frac{x^2+5x+5}{x^3+4x^2+5x} dx = \int \frac{x^2+4x+5+x}{x(x^2+4x+5)} dx$$

$$= \int \left[\frac{1}{x} + \frac{1}{x^2+4x+5} \right] dx \quad (\text{on fact. spl})$$

$$= \ln|x| + \int \frac{1}{(x^2+4x+4)+1} dx$$

$$= \ln|x| + \int \frac{1}{(x+2)^2+1} dx$$

$$= \ln|x| + \arctan(x+2) + C$$

$$6) \int \ln(1+x^2) dx$$

$$\begin{cases} f = \ln(1+x^2) \\ g' = 1 \end{cases}$$

$$\begin{cases} f' = \frac{2x}{x^2+1} \\ g = x \end{cases}$$

$$I = x \ln(1+x^2) - 2 \int \frac{x^2}{x^2+1} dx$$

$$= x \ln(1+x^2) - 2 \int \left[1 - \frac{1}{x^2+1} \right] dx$$

div. euclidiana

$$= n \ln(n^2 + 1) - 2n + 2 \text{ constant } n + C \quad (6)$$

$$2) I = \int \left(1 - \frac{1}{n}\right) e^{\frac{1}{n}} dn$$

On pose $t = \frac{1}{n} \Leftrightarrow n = \frac{1}{t} \Leftrightarrow dn = -\frac{1}{t^2} dt$

$$I = \int \left(1 - t\right) e^t \frac{1}{t^2} dt$$

$$= \int \frac{t-1}{t^2} e^t dt = \int \frac{e^t}{t} dt - \underbrace{\int \frac{e^t}{t^2} dt}_{I_2}$$

$$I_2 = \int \frac{e^t}{t^2} dt$$

$$\begin{cases} f = e^t \\ g' = \frac{1}{t^2} = t^{-2} \end{cases} \quad \begin{cases} f' = e^t \\ g = \frac{t^{-1}}{-1} = -\frac{1}{t} \end{cases}$$

$$I_2 = -\frac{e^t}{t} + \int \frac{e^t}{t} dt$$

$$I = \cancel{\int \frac{e^t}{t} dt} - \left[-\frac{e^t}{t} + \cancel{\int \frac{e^t}{t} dt} \right]$$

$$= \frac{e^t}{t} + C$$

$$= n e^{\frac{1}{n}} + C$$

Nombres complexes

$$1) z^2 = (2+3i)^2 = -5+12i$$

$$z^4 = (-5+12i)^2 = -119-120i$$

$$\begin{aligned} \Rightarrow E(z) &= \frac{-5+12i+2+3i+1}{-119-120i-1} = \frac{-2+15i}{-120(1+i)} \quad \frac{1-i}{1-i} \\ &= \frac{13+17i}{-120 \cdot 2} = -\frac{13+17i}{240} \end{aligned}$$

$$\begin{aligned} 2) z^3 &= (-1-i)^3 = -(1+3i+3i^2+i^3) \\ &= -(1+3i-3-i) \\ &= -(-2+2i) \\ &= 2-2i \end{aligned}$$

$$\begin{aligned} z^5 &= z^3 z^2 = (2-2i)(-5+12i)^2 \\ &= (2-2i)2i \\ &= 4+4i \end{aligned}$$

$$\begin{aligned} E(z) &= \frac{2-2i-1}{4+4i-1} = \frac{1-2i}{3+4i} \cdot \frac{3-4i}{3-4i} \\ &= \frac{-5-10i}{9+16} = -\frac{1+2i}{5} \end{aligned}$$

$$\begin{aligned} 3) (a+bi)^3 &= a^3 + 3a^2bi + 3a(bi)^2 + (bi)^3 \\ &= a^3 + 3a^2bi - 3ab^2 - b^3i \end{aligned}$$

$$(a+bi)^3 \text{ réel} \Leftrightarrow 3a^2b - b^3 = 0$$

$$\Leftrightarrow b(3a^2 - b^2) = 0$$

$$\Leftrightarrow \begin{cases} b = 0 \\ b = \pm \sqrt{3}a \end{cases}$$

$$4) j = -\frac{1}{2} + i \frac{\sqrt{3}}{2}$$

⑦

$$a) j^2 = \left(-\frac{1}{2} + i \frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} i - \frac{3}{4}$$

$$= -\frac{1}{2} - i \frac{\sqrt{3}}{2} = \bar{j}$$

$$b) \frac{1}{j} = j^2 \Leftrightarrow \frac{1}{j} = \bar{j} \Leftrightarrow j \bar{j} = 1$$

$$\Leftrightarrow \left(-\frac{1}{2} - i \frac{\sqrt{3}}{2}\right) \left(-\frac{1}{2} + i \frac{\sqrt{3}}{2}\right) = 1$$

$$\Leftrightarrow \frac{1}{4} - i^2 \frac{3}{4} = 1 \quad (\text{on } i^2 = -1)$$

$$c) \frac{1}{j} = j^2 \Leftrightarrow j^3 = 1 \Leftrightarrow j^3 - 1 = 0$$

$$\Leftrightarrow (j-1)(j^2+j+1) = 0 \Leftrightarrow j^2+j+1 = 0$$

$\swarrow \neq 0$

$$d) \frac{1}{1+j} = \frac{1}{\frac{1}{2} + i \frac{\sqrt{3}}{2}} \cdot \frac{\frac{1}{2} - i \frac{\sqrt{3}}{2}}{\frac{1}{2} - i \frac{\sqrt{3}}{2}}$$

$$= \frac{\frac{1}{2} - i \frac{\sqrt{3}}{2}}{\frac{1}{4} + \frac{3}{4}} = \frac{1}{2} - i \frac{\sqrt{3}}{2}$$

5) Soient $z = a + bi$ et $z' = a' + b'i$

$$a) z + z' = (a + a') + i(b + b')$$

$$\overline{z + z'} = (a + a') - i(b + b')$$

$$\text{et } \bar{z} + \bar{z}' = (a - bi) + (a' - b'i)$$

$$= (a + a') - i(b + b')$$

$$b) z z' = (aa' - bb') + i(ab' + a'b)$$

$$\overline{z z'} = (aa' - bb') - i(ab' + a'b)$$

$$\bar{z} \cdot \bar{z}' = (a - bi)(a' - b'i)$$

$$= (aa' + bb') - i(ab' + a'b)$$

6) a) Soit $u = x + iy$ la racine de $-2 + \frac{3i}{2}$
 On a $\begin{cases} x^2 - y^2 = -2 \\ 2xy = \frac{3}{2} \end{cases} \Leftrightarrow \begin{cases} x^2 - \frac{9}{16x^2} = -2 \\ y = \frac{3}{4x} \end{cases} (*)$

$(*) \quad 16x^4 + 32x^2 - 9 = 0$
 $\Delta = 1600 \Rightarrow x_{1,2}^2 = \frac{-32 \pm 40}{32} \begin{cases} \frac{1}{4} \\ -\frac{13}{4} \end{cases} \text{ AR}$

$\Rightarrow x = \pm \frac{1}{2} \Leftrightarrow y = \frac{\pm 3}{4 \cdot \frac{1}{2}} = \pm \frac{3}{2}$

$\Rightarrow u = \pm \left(\frac{1}{2} + \frac{3i}{2} \right)$

b) $\begin{cases} x^2 - y^2 = -15 \\ 2xy = -8 \end{cases} \Leftrightarrow \begin{cases} x^2 - \frac{16}{x^2} = -15 \\ y = -\frac{4}{x} \end{cases} (*)$

$(*) \quad x^4 + 15x^2 - 16 = 0$
 $\Delta = 289 \quad x_{1,2}^2 = \frac{-15 \pm 14}{2} \begin{cases} 1 \\ -16 \end{cases} \text{ AR}$

$x = \pm 1 \Rightarrow y = \mp 4$

$\Rightarrow u = \pm (1 - 4i)$

7) a) $z^2 = 5 + 12i \Rightarrow$ on cherche les racines de $5 + 12i$ (cf en. 6)

$\Rightarrow z = \pm (3 + 2i)$

b) $\Delta = (20 + 9i)^2 - 400$
 $= 400 + 360i - 81 - 400$
 $= 360i - 81$

les racines de Δ sont $12 + 15i$

et $z_{1,2} = \frac{20 + 9i \pm (12 + 15i)}{4} \begin{cases} 8 + 6i \\ 2 - \frac{3i}{2} \end{cases}$

$$c) \Delta = (3-i)^2 - 8(1-i)$$

$$= \cancel{9} - \cancel{1} - 6i - \cancel{8} + 8i$$

$$= 2i$$

⑧

les racines de Δ sont $1+i$

$$\text{et } z_{1,2} = \frac{3-i \pm (1+i)}{2(1-i)}$$

$$z_1 = \frac{2}{1-i} \cdot \frac{1+i}{1+i} = \frac{2(1+i)}{2} = 1+i$$

$$z_2 = 1$$

$$d) z^2 - (3+5i)z - (4+17i) = 0$$

$$\Delta = (3+5i)^2 + 4(4+17i)$$

$$= 9 - 25 + 30i + 16 + 68i$$

$$= 98i$$

les racines de Δ sont $7+7i$

$$z_{1,2} = \frac{3+5i \pm (7+7i)}{2} \begin{cases} 5+6i \\ -2-i \end{cases}$$

8) a) $z = a$ est racine si $P(a) = 0$

$$\Leftrightarrow a^3 - 8a^2 + 25a - 26 = 0$$

$$P(2) = 0 \Rightarrow a = 2$$

$$b) \begin{array}{c|ccc} & 1 & -8 & 25 & -26 \\ \hline 2 & & 2 & -12 & 26 \\ \hline & 1 & -6 & 13 & 0 \end{array}$$

$$P(z) = (z-2)(z^2 - 6z + 13)$$

$$\Leftrightarrow \Delta = -16$$

$$z_{1,2} = \frac{6 \pm 4i}{2} = 3 \pm 2i$$

$$\Rightarrow P(z) = (z-2)(z-3+2i)(z-3-2i)$$

9) a) Comme pour le 8) $P(bi) = 0$

$$\Leftrightarrow (bi)^3 - (5+ci)(bi)^2 + (1-33i)(bi) - 290 - 30i = 0$$

$$\Leftrightarrow 5b^2 + 33b - 290 - i(b^3 - 6b^2 - b + 30)$$

$$\Leftrightarrow \begin{cases} 5b^2 + 33b - 290 = 0 & (1) \\ b^3 - 6b^2 - b + 30 = 0 & (2) \end{cases}$$

$$(1) \Rightarrow b = -\frac{58}{5} \text{ ou } b = 5 \quad (\Delta)$$

Seule la solution $b = 5$ vérifie l'éq (2)

$$b) \begin{array}{c|ccc} & 1 & -5-6i & 1-33i & -290-30i \\ \hline 5i & 5i & 5-25i & 290+30i & \\ \hline & 1 & -5-i & 6-58i & 0 \end{array}$$

$$P(z) = (z - 5i)(z^2 - (5+i)z + 6-58i)$$

$$\Delta = 242i$$

Les racines de Δ sont $11(1+i)$

$$z_{1,2} = \frac{5+i \pm (11+11i)}{2} \begin{cases} 8+6i \\ -3-5i \end{cases}$$

$$P(z) = (z - 5i)(z - 8 - ci)(z + 3 + 5i)$$

10) $z_1 = \sqrt{3} - i = 2 \cos\left(-\frac{\pi}{6}\right)$

$$z_2 = 1 - i = \sqrt{2} \cos\left(-\frac{\pi}{4}\right)$$

$$z_3 = i = \cos \frac{\pi}{2}$$

$$z_4 = 1 + i = \sqrt{2} \cos \frac{\pi}{4}$$

$$\frac{(\sqrt{3} - i)^3 (1 - i)^4}{i^7 (1 + i)^6} = \frac{2^3 \cos\left(-\frac{\pi}{2}\right) (\sqrt{2})^4 \cos(-\pi)}{\cos \frac{7\pi}{2} \cdot (\sqrt{2})^6 \cos \frac{3\pi}{2}}$$

$$= 4 \cos\left(-\frac{\pi}{2} - \pi - \frac{7\pi}{2} - \frac{3\pi}{2}\right)$$

$$= 4 \cos\left(-\frac{13\pi}{2}\right) = -4$$

11) a) $z^4 = 7 - 24i$

(9)

$$7 - 24i \Rightarrow \rho = 25, \quad \theta = \arctan\left(-\frac{24}{7}\right) \approx -1,287 \text{ rad } \left(\frac{11\pi}{12}\right)$$

$$\begin{aligned} \Rightarrow z &= \sqrt[4]{25} \operatorname{cis}\left(-\frac{1,287}{4} + \frac{2k\pi}{4}\right) \Rightarrow 7 - 24i = 25 \operatorname{cis}(-1,287) \\ &= \sqrt{5} \operatorname{cis}\left(-0,322 + \frac{k\pi}{2}\right) \end{aligned}$$

b) $z^6 = \frac{\sqrt{3}}{2} + \frac{1}{2}i$

$$\begin{aligned} \frac{\sqrt{3}}{2} + \frac{1}{2}i &\rightarrow \rho = 1, \quad \theta = \frac{\pi}{6} \\ \Rightarrow \frac{\sqrt{3}}{2} + \frac{1}{2}i &= \operatorname{cis} \frac{\pi}{6} \end{aligned}$$

$$\Rightarrow z = \operatorname{cis}\left(\frac{\pi}{36} + \frac{2k\pi}{6}\right)$$

c) On pose $x = z^3$

$$\Rightarrow x^2 + (2i - 1)x - 1 - i = 0$$

$$\Delta = 1 \Rightarrow x_{1,2} = \frac{-2i + 1 \pm 1}{2} \begin{cases} -i + 1 \\ -i \end{cases}$$

$$\begin{aligned} \cdot z^3 = -i + 1 &\quad \text{et } -i + 1 = \sqrt{2} \operatorname{cis}\left(-\frac{\pi}{4}\right) \\ \Rightarrow z &= \sqrt[3]{2} \operatorname{cis}\left(-\frac{\pi}{12} + \frac{2k\pi}{3}\right) \end{aligned}$$

$$\begin{aligned} \cdot z^3 = -i &\quad \text{et } -i = \operatorname{cis}\left(-\frac{\pi}{2}\right) \\ \Rightarrow z &= \operatorname{cis}\left(-\frac{\pi}{6} + \frac{2k\pi}{3}\right) \end{aligned}$$

Représentation $\rightarrow$ voir annexe

