Exercices complémentaires : Etudes de fonctions logarithmiques et exponentielles : Solutions

Faire l'étude complète (~~pas la dérivée seconde~~) des fonctions suivantes :

1. $f(x) = \dfrac{\ln x}{x}$

• $\text{dom}_f : \mathbb{R}_0^+$

• zéro : $x = 1$

• $\cap\, Oy$: /

signe : $\circledast$

• AV : $\lim\limits_{0} f(x) = \dfrac{-\infty}{0} = -\infty \Rightarrow AV \equiv x = 0$

AH : $\lim\limits_{+\infty} f(x) = \dfrac{\infty}{\infty}$ FI $\overset{RH}{=} \lim\limits_{+\infty} \dfrac{\frac{1}{x}}{1} = 0$

$\rightarrow AH \equiv y = 0 \Rightarrow \cancel{AO}$

• $f'(x) = \dfrac{\frac{1}{x} \cdot x - \ln x}{x^2}$

$= \dfrac{1 - \ln x}{x^2}$

$1 - \ln x = 0 \Leftrightarrow x = e$

x		0		e	
$f'(x)$	//////		$+$	0	$-$
$f(x)$	//////		$\nearrow$	M	$\searrow$

$\left(e, \frac{1}{e}\right)$

• $f''(x) = \dfrac{-\frac{1}{\cancel{x}}\, \cancel{x^2} - 2\cancel{x}(1 - \ln x)}{x^{\cancel{4}\,3}}$

$= \dfrac{-3 + 2\ln x}{x^3}$

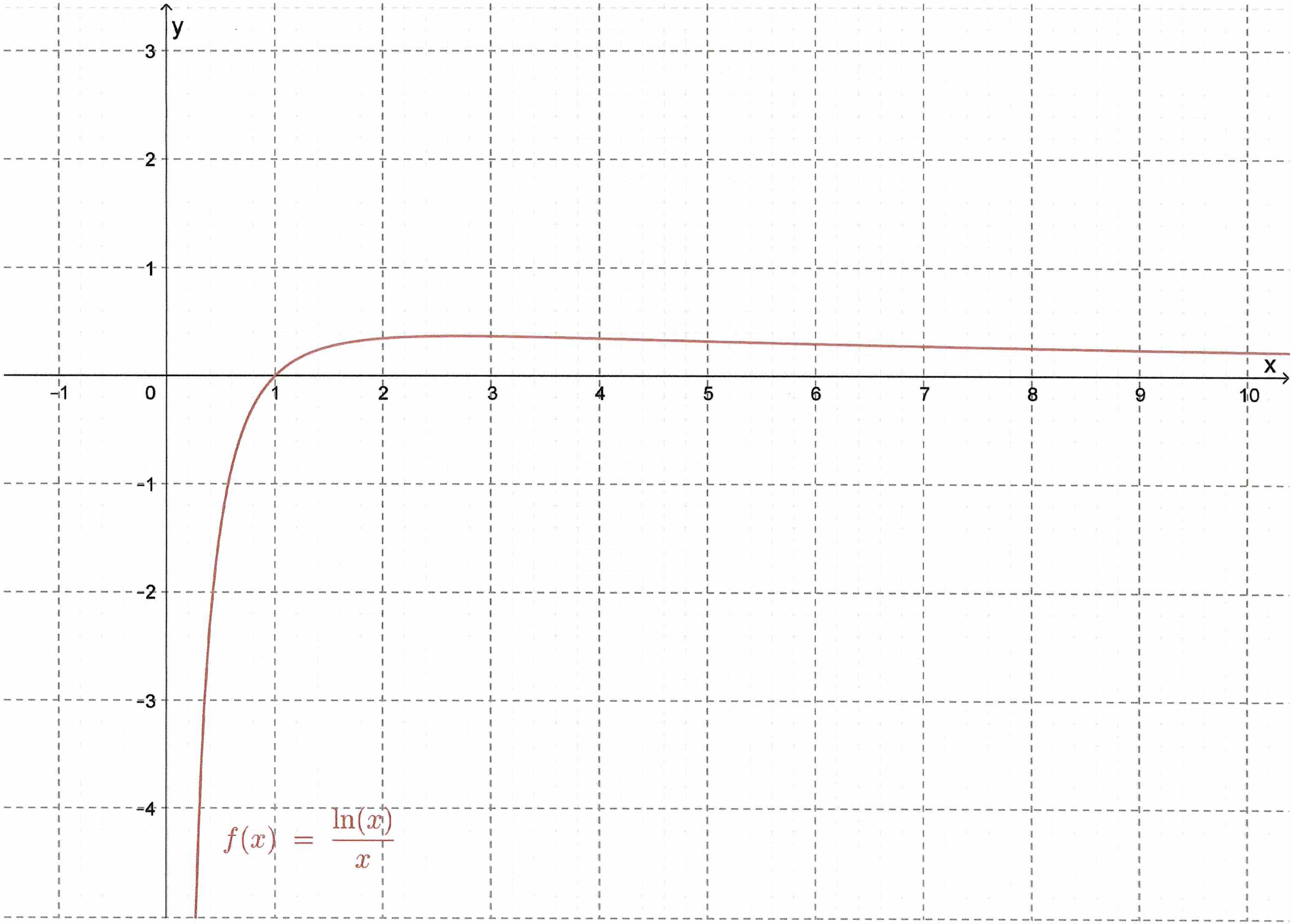
x		0		$e^{\frac{3}{2}}$	
$f''(x)$	//////		$-$	0	$+$
$f(x)$	//////		$\cap$	PI	$\cup$

$\left(e^{\frac{3}{2}}, \dfrac{3}{2e^{3/2}}\right)$

(*) <u>signe</u>

x	0		1	
$\ln x$	||	$-$	0	$+$
x	||	$+$		$+$
$f(x)$	||	$-$	0	$+$

Free Plain Graph Paper from http://incompetech.com/graphpaper/plain/


y
x
-1
0
1
2
3
4
5
6
7
8
9
10
3
2
1
-1
-2
-3
-4
$f(x) = \frac{\ln(x)}{x}$

8. $f(x) = 3e^{2x} - 5e^{x}$

- dom f : $\mathbb{R}$
- zéro : $e^x(3e^x - 5) = 0 \Leftrightarrow e^x = \frac{5}{3} \Leftrightarrow x = \ln\frac{5}{3}$
- $\cap Oy$: $f(0) = -2$
- signe

x		$\ln\frac{5}{3}$	
$f(x)$	$-$	0	$+$

- AV : /

AH : $\lim\limits_{-\infty} f(x) = 0 \to AH_g \equiv y = 0$

$\lim\limits_{+\infty} f(x) = \infty$ (car $e^{2x} > e^x$) $\Rightarrow$ AH /

AO : non car $e^x > x$

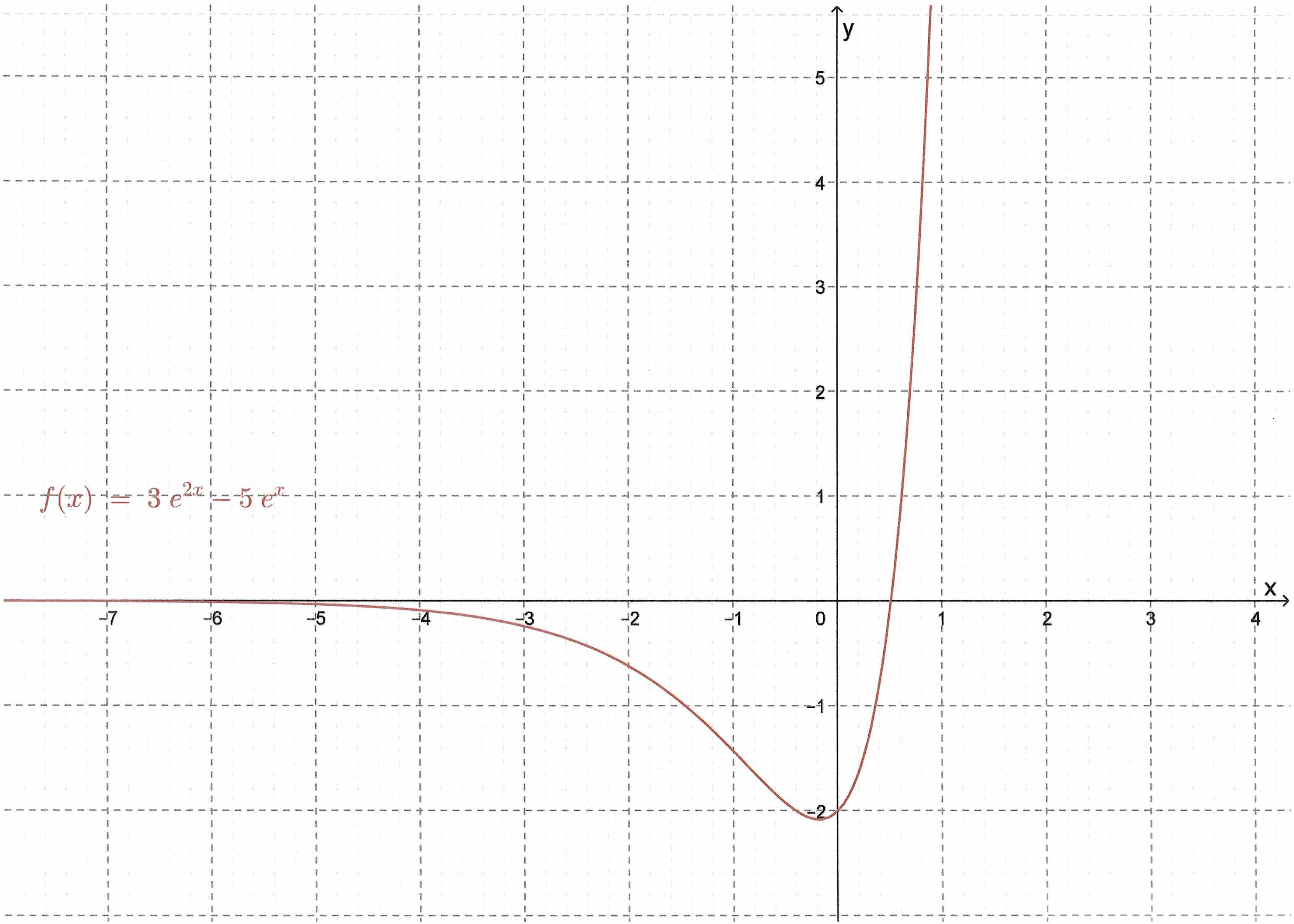
- $f'(x) = 6e^{2x} - 5e^x$ $(= e^x(6e^x - 5))$

x		$\ln\frac{5}{6}$	
$f'(x)$	$-$	0	$+$
$f(x)$	$\searrow$	m	$\nearrow$
		$\left(\ln\frac{5}{6}, -\frac{25}{12}\right)$	

- $f''(x) = 12e^{2x} - 5e^x$

x		$\ln\frac{5}{12}$	
$f''(x)$	$-$	0	$+$
$f(x)$	$\cap$	PI	$\cup$
		$\left(\ln\frac{5}{12}, -\frac{25}{16}\right)$	

Free Plain Graph Paper from http://incompetech.com/graphpaper/plain/



$f(x) = 3\,e^{2x} - 5\,e^{x}$
x
y
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5

17 ~~18.~~ $f(x) = \ln(x^2 - 1)$

. CÉ : $x^2 - 1 > 0 \iff x \in]-\infty, -1[\cup]1, +\infty[$

. zéro : $\ln(x^2 - 1) = 0 \iff x^2 - 1 = 1 \iff x = \pm\sqrt{2}$

. $\cap$ Oy : /

(fct paire)

. signe

x		$-\sqrt{2}$		-1		1		$\sqrt{2}$	
$f(x)$	$+$	0	$-$	$\nexists$		$\nexists$	$-$	0	$+$

. AV : $\lim\limits_{\pm 1} f(x) = \pm\infty \rightarrow AV_1 \equiv x = -1$

$AV_2 \equiv x = 1$

AH : $\lim\limits_{\pm\infty} f(x) = \ln \infty = \infty \Rightarrow$ AH /

AO : $m = \lim\limits_{\pm\infty} \dfrac{f(x)}{x} = \dfrac{\infty}{\infty}$ FI

RH $= \lim\limits_{\pm\infty} \dfrac{\frac{2x}{x^2-1}}{1} = \lim\limits_{\pm\infty} \dfrac{2x}{x^2-1} = 0$

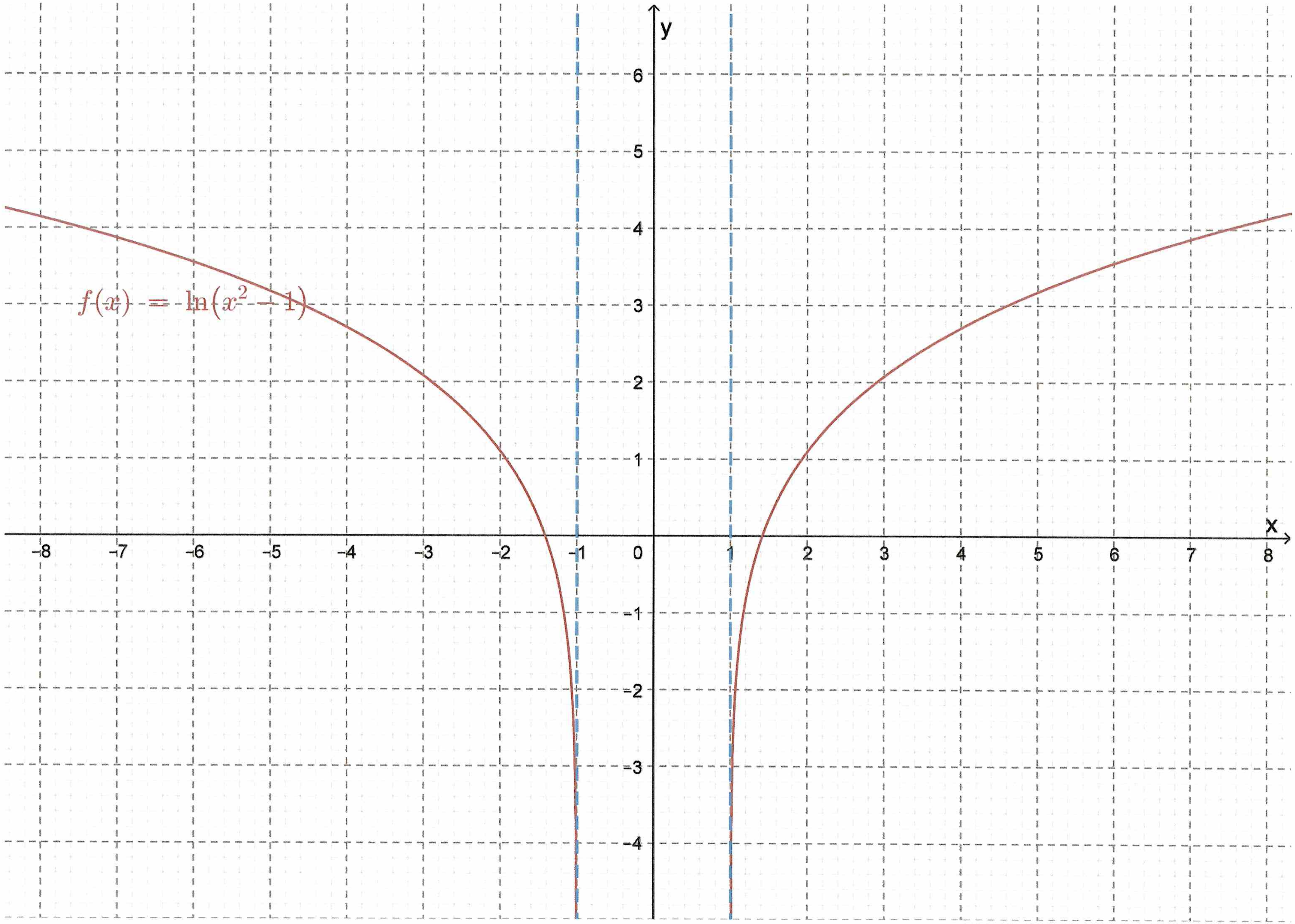
$\rightarrow$ AO /

. $f'(x) = \dfrac{2x}{x^2 - 1}$

x		-1		0		1	
N	$-$		$-$	0	$+$		$+$
D	$+$	0	$-$		$-$	0	$+$
$f'(x)$	$-$	$\nexists$				$\nexists$	$+$
$f(x)$	$\searrow$	AV				AV	$\nearrow$

. $f''(x) = \dfrac{2(x^2-1) - 2x \cdot 2x}{(x^2-1)^2} = \dfrac{-2x^2 - 1}{(x^2-1)^2}$

tjs $< 0 \Rightarrow f(x)$: $\cap$


$f(x) = \ln(x^2 - 1)$
y
x
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
6
5
4
3
2
1
-1
-2
-3
-4

30. $f(x) = x^2 e^x$

. $\text{dom}_f : \mathbb{R}$

. 3 eixo : $x = 0$

. $\cap Oy : y = 0$

. signe : $f(x) \geq 0$

. AV —

AH : $\lim\limits_{-\infty} f(x) = +\infty . 0$ FI

$= \lim\limits_{-\infty} \frac{x^2}{e^{-x}} = \frac{\infty}{\infty}$ FI $\overset{RH}{=} \lim\limits_{-\infty} \frac{2x}{-e^{-x}} = \frac{\infty}{\infty}$ FI

$\overset{RH}{=} \lim\limits_{-\infty} \frac{2}{e^{-x}} = 0 \Rightarrow AH_g \equiv y = 0$

$\lim\limits_{+\infty} f(x) = +\infty \quad AH_d$ —

AO — car $e^x > x$

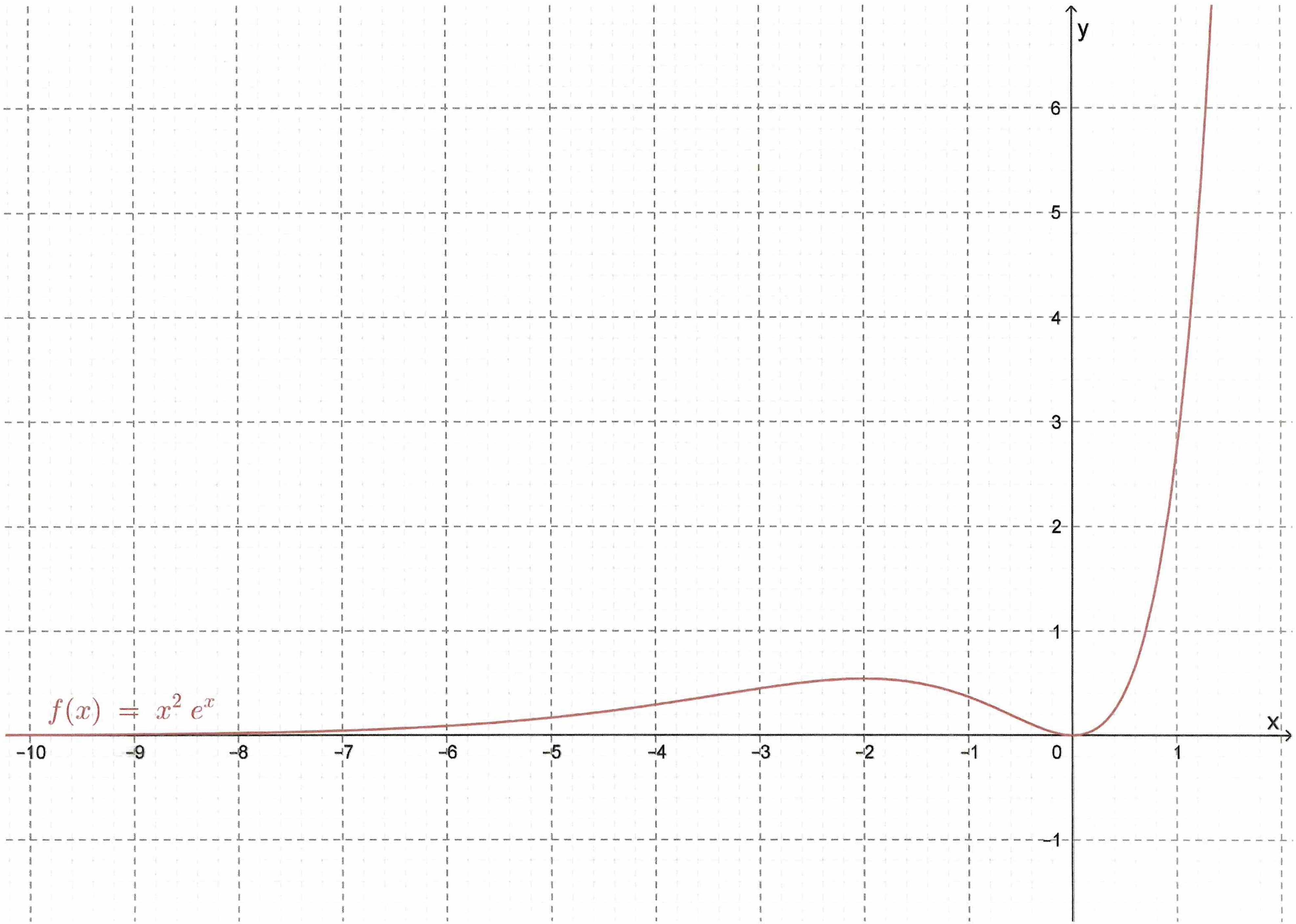
. $f'(x) = 2x e^x + x^2 e^x = (x^2 + 2x) e^x$

x		-2		0	
$f'(x)$	$+$	0	$-$	0	$+$
$f(x)$	$\nearrow$	M	$\searrow$	m	$\nearrow$
		$(-2, \frac{4}{e^2})$		$(0,0)$	

. $f''(x) = (2x+2)e^x + (x^2+2x)e^x = (x^2+4x+2)e^x$

x		$-2-\sqrt{2}$		$-2+\sqrt{2}$	
$f''(x)$	$+$	0	$-$	0	$+$
$f(x)$	$\cup$	PI	$\cap$	PI	$\cup$

$\leftarrow \Delta$



y
x
6
5
4
3
2
1
0
-1
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
1
$f(x) = x^2 e^x$